*Investigating Political Dynamics Using Fractional Integration Methods**

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Theory: Many questions central to political science, such as the issue of stability and change in the United States party system, revolve around the degree of persistence or memory in a political process. Fractional integration techniques, which allow researchers to investigate dynamic behavior that falls between the stationary and integrated alternatives, provide more precise ways to test hypotheses about the degree of persistence than current modeling strategies.
Hypotheses: Choices about the treatment of the time series properties of the data and model specification may influence the substantive conclusions drawn about the dynamics of important political processes.
Methods: Fractional integration methods are discussed and compared with common univariate diagnostic tests. A transfer function model of macropartisanship using fractional integration techniques is contrasted with traditional ARMA and ARIMA methods.
Results: Fractional integration techniques offer a more flexible way to model a time series. Using fractional integration techniques, we find that macropartisanship is dominated by a strong permanent component, but also contains transitory dynamics in response to changes in economic evaluations and a measure of presidential approval. Our empirical work shows the importance of taking seriously the time series properties of data to ensure valid inferences about the dynamics of political processes.

Introduction

Political scientists have long been interested in the way political phenomena change over time. For instance, we have theories of incremental budgeting, dependent development, war cycles, political-business cycles, and periodicity in the United States party system. Implicit in these theories are hypotheses about the dynamic path each process follows as well as its response to shocks from events such as elections, social upheaval, war, international crises, or economic change. In some cases, a theory may predict that the effects of a shock will dissipate quickly. In others, a theory may hold that the impact of a shock will endure for long spans of time.

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A common thread linking such theories is that each is concerned with the degree of persistence in a political process. Persistence is a concept that refers to the rate at which a process moves toward an equilibrium level after being perturbed by a shock. Political phenomena that move quickly toward equilibrium are said to have short memory and low persistence while those that reequilibrate slowly are said to have long memory and high persistence. Since competing theories may have different implications for the degree to which the effects of a shock persist, one way to test rival hypotheses is to investigate the degree of persistence and memory in a time series.

MacKuen, Erikson, and Stimson's (1989) seminal analysis, which delves into the causes and dynamic consequences of shifts in macropartisanship, is a prominent example.[1] Invoking the concepts of persistence and memory, they (1989, 1137) conclude that aggregate "partisanship quickly 'forgets' approval while it evinces a more elephantine memory for previous economic conditions." In addition, they suggest that macropartisanship is characterized by "mid-range dynamics" rather than the long periods of stable partisanship predicted by realignment and party system theories.

In later work, MacKuen, Erikson, and Stimson (1992a) find that macropartisanship is composed of a complex mix of dynamic behavior that they characterize as "qualitatively different" from the behavior of a mean-reverting, stationary time series in which the effects of a shock die out quickly or from an integrated, random walk process in which the effects of a shock are permanent.[2] The question of whether the dynamic behavior of a process is better represented by a model for a stationary or for an integrated time series arises frequently for political scientists (e.g., Alt 1985; Clarke and Stewart 1994; Durr 1993; MacKuen, Erikson, and Stimson 1992b; Nardulli 1995; Norpoth and Yantek 1983; Ostrom and Smith 1993; Rajmaira and Ward 1990; Rasler and Thompson 1985; Whiteley 1986).[3] The issue basically comes down to a question of whether to first difference data thought to be

[1]They define macropartisanship as the aggregate percentage Democratic of all party identifiers.

[2]A time series is a random walk, unit root process if it has a root of unity in its autoregressive polynomial (with all other roots inside the unit circle). The unit root produces a series in which the variance depends on time. The mean of the series will also depend on time if the random walk contains drift. A time series is weakly (covariance) stationary if its mean, variance, and the covariances between any two observations an equal distance apart are time invariant.

[3]Some political scientists consider alternatives involving explosive roots, chaos (Richards 1992), or deterministic trends (Mueller 1970). We do not think explosive or chaotic alternatives characterize many, if any, political processes, and hence limit ourselves to the class of linear time series models. We will, however, investigate whether any of our series are linear functions of deterministic time trends. See Freeman et al. (1997) and Nelson and Kang (1981, 1984) for a discussion of the implications of the treatment of trend properties (difference and trend stationarity) in time series data.

integrated and thereby remove a stochastic trend or whether to assume the data are stationary and use them in their levels form.

Various political scientists have resolved the question of levels versus first differences in different ways. For some, the choice to difference the data leads them to use AutoRegressive Integrated Moving Average (ARIMA) model specifications to test their hypotheses (e.g., Alt 1985; Nardulli 1995; Norpoth and Yantek 1983; Rasler and Thompson 1985; Whiteley 1986). In contrast, authors such as MacKuen, Erikson, and Stimson (1989) use the basic AutoRegressive Moving Average (ARMA) transfer function specification in their work on macropartisanship. By choosing an ARMA specification, the authors implicitly assume their data are stationary.

Both ARMA and ARIMA model specifications and technologies have been useful to political scientists, but the choice between the two is more than a matter of taste. Each of the two specifications can lead to dramatically different conclusions about the dynamic effects of a shock to the political process being studied. That is, the choice of model specification strongly influences the conclusions that a researcher draws. When data are fractionally integrated, an AutoRegressive Fractionally Integrated Moving Average (ARFIMA) model specification becomes an appropriate choice. Models for fractionally integrated data not only help researchers faced with the choice of differencing the data or using it in levels, but ARFIMA models also supply political scientists with another model specification for their toolboxes—one that can shed new and important light on the dynamic behavior of a political process.

But are there theories that would lead political scientists to posit and use an ARFIMA transfer function model specification? The answer is yes. The previous discussion of MacKuen, Erikson, and Stimson's characterization of the memory of partisanship suggests such a process. Given this, it is plausible that other political processes may also exhibit a broader range of decay than could previously be detected by existing methods.

Current theories about why a time series might be fractionally integrated seem just as applicable to political, as to economic, processes. In one theory, Granger (1980) argues that aggregating over heterogeneous micro-level behavioral mechanisms with an autoregressive form results in an aggregate time series that is fractionally integrated. Box-Steffensmeier and Smith (1996) show the applicability of this latter argument to issues in political science by using Granger's aggregation results and assumptions from existing theories of individual party identification to predict patterns in aggregate levels of partisanship. As Box-Steffensmeier and Smith (1996) argue, if current theories of the individual party identification decision are correct, then neither the stationary nor the integrated alternatives will accurately approximate the degree of persistence and memory in aggregate partisanship series. Instead, Box-Steffensmeier and Smith (1996) provide theory and evidence to show that aggregate levels of party identification are fractionally integrated, exhibiting long, but not infinite, memory and persistence.[4]

In a second case, economists argue that certain processes "inherit" their fractional integration properties from the exogenous forces that affect them. Such a theory, which begs the question of why the exogenous variables are themselves fractionally integrated, is used in the literature on multivariate models for fractional cointegration in which certain time series are predicted to move together over time (e.g., Cheung and Lai 1993).

Because political scientists are often interested in the degree of persistence in a political process and because they frequently use ARMA and ARIMA model specifications in their work, it seems important to introduce researchers to the alternative ARFIMA model specification. Furthermore, because theory and evidence show that at least one important political process—macropartisanship—is fractionally integrated, explaining techniques for fractionally integrated data is both timely and substantively relevant for political scientists. Therefore, the purpose of this paper is to introduce political scientists to methods and issues related to fractionally integrated processes as an alternative to the current binary choice between models for stationary versus integrated data. Because methods for fractionally integrated series allow analysts to investigate dynamic behavior that falls between the stationary and random walk alternatives, use of these methods will allow political scientists to develop more precise tests of hypotheses about the degree of persistence or memory in a time series as well as enable them to use appropriate model specifications for their data.

Fractional Integration

Barkoulas and Baum (1997b) define the type of long memory found in fractionally integrated time series as follows: "Long memory, or long term dependence, describes the correlation structure of a series at long lags. If a series exhibits long memory, there is persistent temporal dependence even between distant observations." To model such long-range persistence, analysts can use an ARFIMA model in which the fractionally integrated time series is represented as:

$$\phi(L)(1 - L)^d x_t = \theta(L)\varepsilon_t \qquad [1]$$

[4]Shocks to a fractionally integrated process do not persist infinitely as they do for integrated processes, but also decline at slower rates than they do for stationary processes. Models for fractionally integrated time series were developed by Granger (1980), Granger and Joyeux (1980), and Hosking (1981). See the 1996 special issue of the *Journal of Econometrics*, edited by Baillie and King, that focuses on the topic of fractional differencing and long memory processes.

where d can take noninteger values, ε_t has an unconditional $N(0, \sigma^2)$ distribution, and where $\phi(L)$ and $\theta(L)$ represent stationary autoregressive (AR) and moving average (MA) components, respectively.[5] Time series fitting this definition are called ARFIMA(p,d,q) processes.

Notice that when d = 0 in Equation 1, x_t is a stationary ARMA(p,q) process with a constant mean and variance over time. Such stationary series have short memory and are mean reverting since the correlation between consecutive observations dies out quickly. In contrast, when d = 1 in Equation 1, x_t is a nonstationary, ARIMA(p,1,q) process with a unit root. Such series are labeled integrated because the effects of a shock persist at full force in each period and accumulate over time. Integrated processes have theoretically infinite variances, exhibit long stochastic swings up or down, and do not return to a constant mean level.[6]

Comparisons

Fractionally integrated time series differ from both stationary and integrated processes.[7] Unlike weakly dependent ARMA processes, fractionally integrated time series exhibit significant dependence between observations. Although fractionally integrated processes are mean reverting, the autocorrelation function of a fractionally integrated series is different from that of a stationary autoregressive time series—the former decays at a hyperbolic rate while the latter declines at an exponential rate.

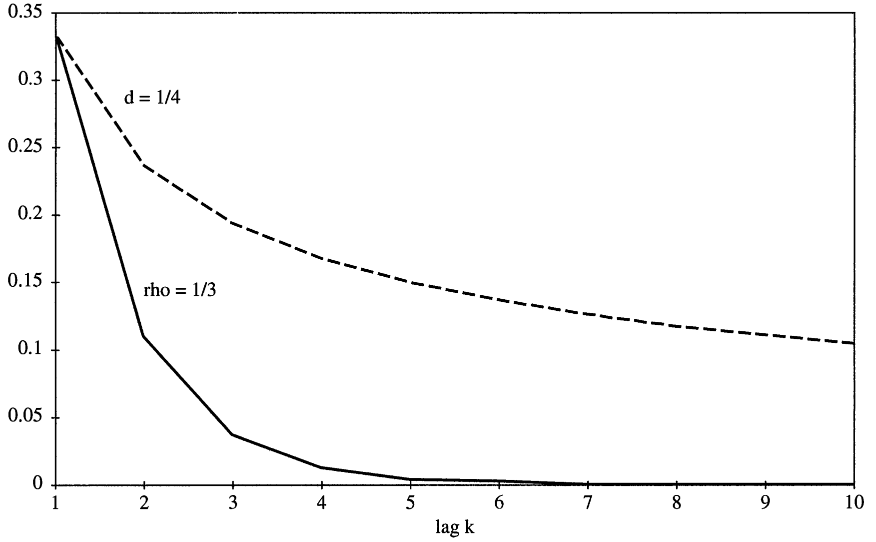
To illustrate these differences, Figure 1 compares the autocorrelation function for a stationary, AR(1) process with $\rho = 1/3$ to the autocorrelation function for a pure fractional noise process with d = 1/4. These two processes are parameterized so that they have the same positive first-order autocorrelation. But notice that as the interval between observations increases, the paths of the autocorrelation functions diverge. By lag 6, the autocorrelation function is approximately zero for the AR(1) process while it is still 0.14 for the fractionally integrated series. This slow, smooth decay of

[5]The symbol, L, represents the lag operator, which is defined such that $Lx_t = x_{t-1}$ and $(1 - L)^d x_t = x_t - x_{t-1} = \Delta x_t$ when d = 1. A fractionally integrated time series with no AR or MA components is called pure fractional noise and is represented as $(1 - L)^d x_t = \varepsilon_t$.

[6]Since integrated time series can be made stationary by differencing them once, they are said to be integrated of order one, denoted I(1). Stationary processes are integrated of order zero, denoted I(0).

[7]Technically, integrated, unit root processes in which d = 1 are a special case of the fractionally integrated class of models. For ease of exposition, we are using the words "integrated" to refer to cases when $|d| = 1$ and "fractionally integrated" to refer to cases in which $0 < |d| < 1$. We use the word "stationary" to refer to covariance stationary time series in which d = 0. Fractionally integrated time series are not covariance stationary since the autocovariances of any two observations the same distance apart can depend on time (as can the variance of the series).

Figure 1. Autocorrelation Functions



the autocorrelation function for fractionally integrated processes has been called its defining characteristic (Lo 1991, 1286).[8]

Fractionally integrated processes are distinct from stationary processes, but also differ from integrated, unit root series. Although fractionally inte-

[8]The autocorrelation function for an AR(1) process with negative values of ρ exhibits the sawtooth pattern associated with negative serial correlation, while the autocorrelation function for a fractionally integrated series with negative values of d declines smoothly over time.

We can also look at the autocorrelation function of a fractionally integrated time series in a more formal manner. First, Hosking (1981, 167), relying on a result from Gradshteyn and Ryzhik (1965), proves that the autocovariance function of a fractionally integrated time series is given by:

$$E(x_t x_{t-k}) = \gamma_k = \frac{(-1)^k(-2d)!}{(k-d)!(-k-d)!} \quad \text{[N1]}$$

This implies that the variance of a fractionally integrated series is:

$$\gamma_0 = \frac{(1)(-2d)!}{(-d)!(-d)!} = \frac{(-2d)!}{[(-d)!]^2} \quad \text{[N2\}}$$

Thus, forming the autocorrelation function (ACF) in the usual way, we see that the ACF for a fractionally integrated series is given by:

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \frac{(-d)!(k+d-1)!}{(d-1)!(k-d)!} \quad \text{[N3]}$$

As shown in Equation N3, the ACF for a fractionally integrated series decays in a hyperbolic fashion according to a hypergeometric function rather than in the exponential manner of a stationary time series. In Appendix A we derive the ACF for a fractionally integrated series and prove that the expression in Equation N3 is true.

Table 1. Time Series Characteristics

	d = 0	0 < d < 0.5	0.5 ≤ d < 1	d = 1
Series type	stationary	fractionally integrated		integrated
Memory	short	long		infinite
Mean reversion	yes	yes		no
Variance	finite	finite	infinite	infinite

grated processes are characterized by persistence and long memory, the effects of a shock do not persist at full force in each period—the memory of a fractionally integrated series is long, but not infinite. Ultimately, the effects of a shock to a fractionally integrated series dissipate as the series reverts to its mean level. Table 1 summarizes the distinctions between stationary, fractionally integrated, and integrated time series.[9] Notice that for smaller values of d, a fractionally integrated series shares some features of a stationary series while for larger values of d, a fractionally integrated series has some of the characteristics of an integrated series. The closer the absolute value of d is to one, the longer the memory and the more persistent are the effects of shocks.

Advantages and Disadvantages

One attraction of models for fractionally integrated time series is that they allow analysts to estimate the order of integration, denoted d. Estimation of d allows analysts to investigate a continuum of dynamic behavior rather than just the discrete d = 0 or d = 1 alternatives, and researchers can do so via parsimonious estimation of a single parameter. In addition, researchers can estimate the standard error of d and obtain information about the degree of uncertainty for the estimate. Estimates of d supply analysts with an objective measure of the properties of a time series rather than the

[9]Some may wonder how "near-integrated" series fit into the classification in Table 1. The answer given by some economists is that they do not, while others contend that it remains an open question. A near integrated series is I(k) with $k = 1 - c/T$ where T is sample size and c is the "noncentrality" parameter. As c changes, the series is characterized either by integration or some local alternative of either strongly autoregressive or mildly explosive near-integration. In contrast, a fractionally integrated series is I(d) with $d < 1$. According to Maddala, the ideas of near integration and fractional integration are not really related. The concept of near integrated series is used to discuss what is known as local to unity asymptotics in discussions of unit root processes (G.S. Maddala, September 11, 1996, personal e-mail communication). See DeBoef et al. (1996); DeBoef, Baillie, and Granato (1997); DeBoef and Granato (1996, 1997) for a thorough discussion of near integrated series.

subjective information obtained from visual inspection of the autocorrelation function or from often contradictory diagnostic tests.

Baillie (1996) summarizes the advantages of estimating d and using the ARFIMA model specification in terms of the ability to avoid knife-edged decisions between stationarity and unit roots and the capacity to model slower rates of decay than would be the case with ARMA, ARIMA, or typical autoregressive distributed lag models. He also cites the theoretical connection between aggregation results and ARFIMA models as an important innovation. Because of these advantages, the techniques associated with fractionally integrated time series are relevant both to political scientists who estimate structural equation models and to those who use ARMA and ARIMA cross correlation or transfer function approaches. For instance, estimates of d and its standard error bear directly on the question of whether a series needs differencing. These methods can also be used to diagnose the properties of the residual series from an OLS regression.[10]

There are important disadvantages to disregarding the existence of fractional integration. Ignoring the possibility of fractional integration threatens the ability of analysts to draw valid inferences about dynamic political behavior. For example, when an analyst differences a fractionally integrated series by imposing $d = 1$, the resulting series will be overdifferenced, a large moving average component will be induced by the inappropriate unit root restriction, and estimates of the remaining short-run effects will be biased.[11] Relatedly, Sowell (1992a) argues that if a stationary ARMA(p,q) model is applied to a fractionally integrated process, invalid inferences will be drawn because there is no way that the AR or MA parameters can capture the effects of both long and short memory.

Although some political scientists have been taught to think of moving average components as capturing the effects of short-run shocks and autoregressive components as capturing longer run effects, both MA and AR processes decay exponentially and are thus short memory (high frequency)

[10] Yajima (1988) and Fox and Taqqu (1986) provide regularity conditions that ensure the asymptotic consistency and normality of the OLS estimator in a regression with fractionally integrated disturbances. However, these results apply only to those cases in which the disturbances are fractionally integrated such that $0 < d < 1/2$. Thus, knowing whether and the degree to which regressors or the disturbances in a regression are fractionally integrated is important to ensure valid inference.

[11] An important point to note is that standard computer programs compute the autocorrelation function for covariance stationary series in terms of rho, whereas a fractionally integrated time series has an ACF that is computed as in footnote 8. First differencing a fractionally integrated time series will result in overdifferencing that will show up regardless of which computation of the ACF is used. However, the exact degree of overdifferencing may look different depending upon which formulation of the ACF is used. This is an area of continued research. The basic issue is one of the relationship between d and rho at all points along the [–1,1] continuum, and this relationship has yet to be precisely worked out by econometricians.

processes. In addition to the fact that such parameters cannot capture both long and short memory, Sowell argues that any AR or MA parameter that does capture long-run behavior automatically imposes restrictions on the types of short-run behavior that can be detected. Sowell (1992a) argues there is no way to pinpoint the fit of an AR or MA parameter to the long-run features of a time series even if that is what the analyst wants to study.[12] Thus, ignoring fractional integration can lead to incorrect conclusions about the effects of shocks to a series and to incorrect long-run predictions about the behavior of a political process.

While techniques for fractionally-integrated time series have methodological advantages, they also have limitations. No one has yet developed a proof of the asymptotic consistency and normality of the OLS estimator when the *levels* of fractionally integrated series for which $.5 \leq d < 1$ are used as regressors or when the order of integration of a regression's disturbances are in the (.5,1) interval. In addition, econometricians have been slow to develop multivariate estimators for fractionally integrated processes, focusing instead on estimating and interpreting cumulative impulse response functions rather than structural coefficients. The literature on multivariate estimation using fractionally integrated data is, however, starting to grow. For instance, there is a solid set of literature on multivariate techniques in the presence of fractional cointegration (e.g., Baillie and Bollerslev 1994; Barkoulas, Baum, and Oguz 1997; Cheung and Lai 1993; Dueker and Asea 1997; Dueker and Startz 1997; see also the review in Baillie 1996). In addition, estimation of transfer function models and cross-correlation functions are widely accepted time series methods into which researchers can incorporate information about the degree of fractional integration. We will provide estimates of an ARFIMA transfer function in the empirical example in this paper.

Testing for Fractional Integration

Before turning to multivariate estimation of ARFIMA transfer functions, we begin with a discussion of univariate tests for fractional integration. These tests are of two types—point estimates of the order of integration and its standard error and diagnostic tests. While the diagnostic tests can shed some light on matters of long versus short memory, they cannot directly pinpoint the degree of persistence in a series. Because point estimates of d and its standard error allow for direct statistical inference and

[12]Intuition suggests that Sowell's arguments also apply to structural and nonstructural autoregressive distributed lag models. In general, ARFIMA models are described as a flexible and parsimonious way to model both the short- and long-term behavior of a time series (Barkoulas and Baum 1997b, 3).

hypothesis testing, we place a greater emphasis on estimation of these parameters. In doing so, we follow the standard practice in the existing econometric literature (e.g., Barkoulas and Baum 1997c; Cheung and Lai 1992; Diebold and Rudebusch 1989; Sowell 1992a, 1992b).

Estimating the Order of Integration

Point estimates of d and its standard error allow analysts to perform hypothesis tests and draw statistical inferences about the degree of persistence in a political process. While both frequency and time domain estimators have been developed to obtain these estimates, Monte Carlo results suggest that maximum likelihood time domain estimators are better for small to medium size samples. And of the ML estimators, Sowell's (1992a) full information exact maximum likelihood time domain estimator, which allows for joint estimation of the long memory parameter, d, along with any short-run AR or MA components, has become the industry standard and is now incorporated into computer programs such as OX, GAUSS, and RATS.[13]

Sowell's ML estimator takes as given a stationary time series, x_t, which follows a fractionally-integrated process as in Equation 1 with $d < .5$[14] and with the roots of $\phi(L)$ simple.[15] Provided these conditions hold and assuming normality, the likelihood function of the ARFIMA(p,d,q) process for $X_T = \{x_1, x_2, \ldots, x_T\}$, a sample of T observations, is given by:

$$L\left(X_T|\Sigma\right) = (2\pi)^{-T/2}\,|\Sigma|^{-1/2}\,e^{\left(-\frac{1}{2}x'\Sigma^{-1}x\right)} \qquad [2]$$

where Σ, is the $T \times T$ Toeplitz autocovariance matrix of X_T, each element of which is a complicated function of d, $\phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q$, and σ^2. See Sowell (1992a, 171–5) for the derivation of these autocovariances. Dahlhaus (1988, 1989) presents sufficient conditions for the consistency and asymptotic normality of the exact ML estimator.

In practice, Sowell (1992a, 171) notes that ML estimation involves writing the autocovariance function in terms of the parameters of the model. This is accomplished by specifying the spectral density of x_t in terms of the

[13]OX, which we used to obtain our estimates of d, is part of the PcGive 9.0 package and is also available free from http://www.eur.nl/few/ei/faculty/ooms/index.html#programs. Chung (1994) offers GAUSS code for ARFIMA(p,d,q) models free to those who contact him at the Department of Economics, Michigan State University. RATS code for certain ARFIMA(p,d,q) processes is available at http://ww2.hawaii.edu/~dmontgom/ (written by David Montgomery) and from ESTIMA at http://www.estima.com. Based on completeness and ease of use, we recommend the OX program.

[14]To ensure $d < 0.5$ and mean reversion, analysts should first difference each data series. The estimate of the order of integration for the level of each series is then equal to 1 plus the estimate of the order of integration for the first differenced data.

[15]Although this assumption is needed for Sowell's proofs, his simulations show that relaxing it has no detrimental effects on parameter estimates.

model's parameters. Sowell says that the spectral density is calculated in two steps. Upon completion of these steps, the autocovariances are then esti mated by integrating over the spectral densities, and then the maximization can occur and the value of the likelihood function can be obtained.[16] The standard errors for d are taken as the square roots of the diagonal elements of the Hessian matrix.

Because Sowell's exact time domain ML estimation approach involves repeated inversion and evaluation of the $T \times T$ autocovariance matrix, Σ, it can be computationally difficult to use in large samples. Fox and Taqqu (1986) develop a frequency-domain ML estimator that is asymptotically equivalent to the exact time domain ML estimator. The Monte Carlo evidence of Cheung and Diebold (1994) shows that time domain ML is more efficient relative to frequency domain ML in small samples, but that the greater efficiency of the time domain ML estimator declines as the sample size grows.[17] Given the Cheung and Diebold (1994) results and our sample size of 160 observations, we chose Sowell's MLE over frequency domain estimation.[18]

Diagnostic Tests for Fractional Integration

Baillie, Chung, and Tieslau (1996, 27) argue that by investigating the pattern of rejections that results from using tests for unit roots in conjunction with tests for stationarity and strong mixing, analysts can obtain information about whether a time series is likely to be fractionally integrated. For instance, rejection of both the null of stationarity (i.e., strong mixing) and the null of a unit root is consistent with the hypothesis that the process under investigation is fractionally integrated. Thus, they argue, diagnostic tests can

[16]More specifically, Sowell (1992a, 175) describes the exact ML estimation of a univariate ARFIMA model as proceeding by factoring the autoregressive polynomial, calculating the autocovariances based on the form of the spectral density of x_t as a function of d and the AR and MA parameters, evaluating the autocovariance matrix, calculating the Cholesky decomposition and determinant of the inverse of the autocovariance matrix, and then calculating the log likelihood function value.

[17]Cheung and Diebold's (1994) evidence pertains to the case in which the mean is unknown and must be estimated. Initial Monte Carlo evidence in Sowell (1992a) showed that when the mean of the process is known, time domain ML is preferred to frequency domain ML estimation (based on a smaller MSE). In practice, the mean of the process must be estimated.

[18]One reviewer wondered whether political scientists have long enough data series to obtain estimates of the degree of long memory or fractional integration in a series. The issue of sample size when estimating d is primarily one of theoretical fit. Since d captures the long-cycle characteristics of a series, a researcher must have a time series at least as long as the theoretically expected cycle. For instance, in the case of macropartisanship, realignment theorists would posit a theoretical cycle of about 35 years, so our sample of 40 years is long enough to include this cycle, should it exist. See Box-Steffensmeier and Smith (1996) for evidence that such a cycle does not exist in macropartisanship.

provide analysts with information about whether a time series is "closer" to being stationary with d = 0 or integrated with d = 1. Unfortunately, some time series are composed of multiple dynamic patterns, and the results from these diagnostic tests do not always point to the same conclusion. Hence, we contend, point estimates of d are more useful (see also Barkoulas and Baum 1997a, whose findings lead them to the same conclusion).

In Table 2, we list the features of five commonly used diagnostic tests. The first of these is the familiar Dickey-Fuller or Augmented Dickey-Fuller test of the null hypothesis of a unit root. The second is the Dickey-Fuller joint F test of the null of a unit root and no time trend. Since each of these tests have low power in the face of fractionally integrated alternatives, researchers who use them may incorrectly fail to reject the null hypothesis of a unit root when a time series is fractionally integrated. Analysts can counter this tendency in various ways.

First, researchers can use a variance ratio test of the null hypothesis of a unit root versus the alternative of pure fractional noise to investigate fractional integration (Diebold and Rudebusch 1991). The intuition behind the variance ratio tests is as follows: When a time series contains a unit root, its variance grows linearly over time. Thus, for a unit root process, k periods multiplied by the variance from period 1 should be equal to the variance in the kth period. Deviations of the variance-ratio statistic from a value of 1.0 indicate departures from the unit root hypothesis.[19]

Second, analysts can test the null hypothesis that a time series is a strong mixing, stationary process.[20] Because fractionally integrated series are not strong mixing, rejection of the null hypothesis provides evidence of fractional integration. At least two important tests of the null hypothesis of strong mixing have been discussed—Lo's (1991) modified rescaled range (R/S) statistic and the KPSS statistic developed by Kwiatkowski, Phillips, Schmidt, and Shin (1992).[21] Both statistics provide information about the degree of long-run dependence in a series by estimating a ratio of a demeaned partial sum process to a consistent estimate of either the "long-run

[19]Diebold (1989) provides critical values and power results for the variance ratio statistic. An Eviews 2.0 routine for implementing a variance ratio test is available from the Inter-university Consortium for Political Research's publication-related archive under the authors' names and a RATS program is available from Estima at http://www.estima.com.

[20]A time series is strong mixing if the rate at which dependence between past and future observations goes to zero as the distance between them grows is "fast" enough (Lo 1991). Stationary autoregressive series, which decay at an exponential rate, are strong mixing processes while fractionally integrated series, which decay at a hyperbolic rate, and unit root processes, which do not decay, are not strong mixing.

[21]See Ostrom and Smith (1993) for a discussion of the KPSS statistic in a political science application. A RATS program for implementing the KPSS test is available from ESTIMA at http://www.estima.com.

variance" of the series or its square root. The KPSS test also allows analysts to test for the presence of deterministic trends.[22]

The key to the KPSS test is to obtain a consistent estimate, denoted $\hat{s}^2(\ell)$, of the long-run variance, which is constructed from the residuals of the regression of x_t on a constant term for a null of stationarity or on a constant and time trend for a null of trend stationarity. Kwiatkowski et al. (1992) provide a decision mechanism for choosing ℓ, the optimal number of lagged residuals to use, for various sample sizes. In small samples, the KPSS statistic is more appropriate than Lo's modified R/S statistic, which has low power when $T < 250$. Moreover, Lee and Schmidt (1996) find that the KPSS test has similar power to the modified rescaled range statistic in distinguishing stationary from fractionally integrated processes in large samples.

Finally, a once popular, but now widely recognized as problematic, estimator of d is a semiparametric, spectral regression-based estimator developed by Geweke and Porter-Hudak (1983). The GPH estimator of d is obtained by regressing the log periodogram of the first differenced time series on an intercept and on $\ln\left(4\sin^2(\lambda_j/2)\right)$, a regressor composed of only the lowest frequency ordinates of the log periodogram. Geweke and Porter-Hudak (1983) show that β_1, the slope parameter from this periodogram regression, is a consistent estimator of the quantity $(1 - d)$ provided the ε_1 are independent and identically distributed. In recent years, the GPH estimator, has been shown to have serious biases (Agiakloglou, Newbold, and Wohar 1993; Hurvich and Ray 1995). In a survey of methods for fractionally integrated time series, Baillie (1996, 33) concludes that, "overall the consensus of evidence is somewhat negative about semi-parametric estimation."

Long Memory and Political Processes: An Empirical Application

Investigating the degree of memory and persistence in aggregate measures of partisanship is critical to evaluating arguments about the nature of stability and change in the United States party system. In previous studies, analysts typically investigated this issue either by using the data in levels or by first differencing the data and removing its long-run components. For instance, MacKuen, Erikson, and Stimson (1989) model macropartisanship using an ARMA transfer function for stationary data in levels. MacKuen, Erikson, and Stimson (1989, 1138) conclude that changes in macropartisanship "are 'permanent' on a scale of months, not decades." In contrast, Box-

[22]As Freeman et al. (1997) point out, "There is no clear and best way to determine whether a time series has a deterministic vs. stochastic trend. The respective tests have low power, and, in small samples such tests are very fragile (Dejong et al. 1992)" (1997, 12). Because we have no theoretical reasons to believe that macropartisanship, consumer sentiment, or presidential approval are a function of deterministic trends (we reject Mueller's 1970 argument), we have additional confidence in our empirical results.

Table 2. Diagnostic Tests for Time Series Characteristics

Test	Formula	Hypotheses	Features—Pros/Cons
DF/ADS test[a]	$\Delta y_t = \alpha + \rho y_{t-1} + \sum_{p=1}^{P} \Delta y_{t-p} + \varepsilon_t$	$H_0: \rho = 1\ (d = 1)$ $H_a: \rho = 0\ (d = 0)$	low power in face of fractionally integrated alternatives
Joint DF/ADF F test[a]	$\Delta y_t = \alpha + \beta t + \rho y_{t-1} + \sum_{p=1}^{P} \Delta y_{t-p} + \varepsilon_t$	$H_0: \rho = 1$ and $\beta = 0$ $H_a: \rho \neq 1$ or $\beta \neq 0$	low power in face of fractionally integrated alternatives
Variance ratio test[b]	$VR(k) = \frac{k\hat{\sigma}_1^2}{\hat{\sigma}_k^2}$ $\hat{\sigma}_k^2 = \frac{1}{T-k+1}\sum_{t=k}^{T}\left(x_t - x_{t-k} - k\hat{\mu}\right)^2$ $\hat{\mu} = \frac{1}{T}\left(x_t - x_{t-1}\right)$	$H_0: d = 1$ $H_a: d < 1$	test for fractional integration

KPSS test[c]	$LM = \frac{1}{T^2}\sum_{t=1}^{T}\frac{S_t^2}{\hat{\sigma}_\varepsilon^2}$ $\hat{\sigma}_e = \hat{s}^2(\ell) = \frac{1}{T}\sum_{t=1}^{T}\hat{e}_t^2 + \frac{2}{T}\sum_{s=\ell}^{\ell} w(s,\ell) \sum_{t=s+1}^{T}\hat{e}_t\hat{e}_{t-s}$ $S_t^2 = \sum_{i=1}^{I}\hat{e}_i^2$	$H_0: d = 0$ $H_a: d = 1$	test null hypothesis of stationarity and strong mixing
GPH test[d]	$\ln\left(I\left(\lambda_j\right)\right) = \beta_0 + \beta_1 \ln\left(4\sin^2\left(\lambda_j/2\right)\right) + \varepsilon_j$ $\beta_1 = (1-d)$	$H_0: \beta_1 = 0$ $H_a: \beta_1 \neq 0$	yields points estimates of d that can be used as ML starting values; estimates can be badly biased in small samples and when series is highly autoregressive

[a]The value of p is the number of lagged first differences it takes to make the residuals of the regression white noise.

[b]Researchers typically report the variance ratio statistic for various values of k between 0 and 32. Diebold (1989) provides power simulations that can help in choosing the most appropriate value of k for various sample sizes.

[c]The residuals come from the regression of y_t on a constant term when the null hypothesis is stationarity. Researchers typically report the KPSS statistic for various value of ℓ. Kwiatkowshi et al. (1992) provide guidance for choosing ℓ.

[d]Geweke and Porter-Hudak (1983) suggest choosing the number of ordinates in the periodogram regression as $T^{.5}$.

Steffensmeier and Smith (1996) explicitly allow for the possibility of fractional integration and conclude that the effects of major shocks to macropartisanship last for years and not months or decades. The different conclusions from these studies could be the result of the differences in choices about model specification.

To illustrate the use of methods for fractional integration and to investigate how choices about the order of integration and model specification affect a researcher's inferences about the degree of persistence in aggregate partisanship, we use a stylized version of the MacKuen, Erikson, and Stimson (1989) first-order transfer function model of macropartisanship.[23] Although MacKuen, Erikson, and Stimson (1989) implicitly assume that macropartisanship is stationary, in their most recent work (Erikson, MacKuen, and Stimson 1996), they hypothesize that macropartisanship is an integrated, random walk process.[24] In fact, Erikson, MacKuen, and Stimson (1996) show that the standard deviation of levels of aggregate partisanship disaggregated by age is increasing over time and thereby violates assumptions of stationarity. Furthermore, Box-Steffensmeier and Smith (1996) argue that macropartisanship is fractionally integrated. Thus, it seems that a reanalysis of MacKuen, Erikson, and Stimson's (1989) work is in order given recent developments in techniques for fractionally integrated time series.[25] The key independent variables in MacKuen, Erikson, and Stimson's (1989) work are citizens' economic evaluations and evaluations of the president's performance.[26] With these measures and some control variables, we will estimate a series of transfer functions using data from the first quarter of 1953 through the fourth quarter of 1992.[27]

[23]For purposes of comparison, we follow MacKuen, Erikson, and Stimson (1989) closely and use the same order transfer function and noise model as they do.

[24]A debate about whether bounded variables can be integrated exists (see DeBoef and Granato 1997; Smith 1993; Williams 1993). We follow Hamilton (1994, 447) and contend that even for bounded data, unit root tests and tests for fractional integration provide useful information about the persistence of a series (see also Box-Steffensmeier and Smith 1996, 572–3).

[25]Green, Palmquist, and Schickler (1996) have also recently reexamined MacKuen, Erikson, and Stimson's results on macropartisanship. Green, Palmquist, and Schickler (1996) primarily emphasize the importance of replication, while we are primarily interested in issues of model specification.

[26]To measure economic evaluations, we follow MacKuen, Erikson, and Stimson (1989) and use the composite Index of Consumer Sentiment. In measuring presidential approval, MacKuen, Erikson, and Stimson (1989, 1140) argue that the "political" portion of a president's approval rating is relevant to movements in macropartisanship. They measure political approval as presidential approval minus "that part of approval forecasted from the economic component alone, with the other parts of the model zeroed out." In practice, they used approval minus 0.29 times lagged consumer sentiment. Based on our analyses, which took the time series properties of the data into consideration, we measured political approval as approval minus 0.297 times lagged consumer sentiment.

[27]The summary statistics for the series are: macropartisanship—mean = 61.27, std. dev. = 4.71; presidential approval—mean = 56.45, std. dev = 12.20; the political portion of presidential ap-

Table 3. Summary of Conclusions from Diagnostic Tests

	Macropartisanship	Consumer Sentiment	Presidential Approval
Joint F test	$d = 1$	$d = 1$	$d = 0$
VR test	$0 < d \leq 1$	$d = 1$	$0 < d \leq 1$
KPSS test for strong mixing	$d > 0$	$d > 0$	$d > 0$
Overall conclusion:	fractionally integrated; not deterministic function of time	fractionally integrated or integrated; not deterministic function of time	stationary or fractionally integrated; not deterministic function of time

Univariate Tests

Although they cannot precisely pinpoint the degree of integration in a time series, we begin our analysis with a brief look at the conclusions we reached on the basis of diagnostic tests of the macropartisanship, consumer sentiment, and presidential approval[28] series. These conclusions appear in Table 3. Both the Dickey-Fuller joint test and the KPSS test for stationarity around a time trend lead us to conclude that none of these series are deterministic functions of a linear time trend.[29] The Dickey-Fuller joint test suggests that presidential approval is stationary while macropartisanship and consumer sentiment are integrated. To counter the low power of the Dickey-Fuller test, we also used: the variance ratio test, which suggested that macropartisanship and presidential approval are not integrated but may be fractionally integrated, and the KPSS test for strong mixing, which suggested that each series is not strong mixing and is likely fractionally integrated. Overall, the evidence from these tests suggests that macropartisanship is fractionally integrated while the evidence about consumer sentiment and presidential approval is mixed.

proval—mean = –13.21, std.dev. = 30.81; and consumer sentiment—mean = 83.30, std. dev. = 14.44. MacKuen, Erikson, and Stimson (1989) used a subset of all Gallup polls available from 1953 to 1988; we do not use this subset but instead the full series of Gallup polls.

[28]Because of the controversy surrounding the properties of presidential approval (e.g., Ostrom and Smith 1993 versus Beck 1993), we emphasize the properties of the untransformed series. The estimate of d for the "political" approval series is .116 with a standard error of .348, so the hypothesis of stationarity cannot be rejected ($t = 0.33$) while the hypothesis of a unit root can be rejected ($t = -2.54$).

[29]With respect to presidential approval, this is an empirical finding with both statistical and substantive implications. For instance, it rules out Mueller's (1970) "coalition of minorities" hypothesis, which he argues will induce a linear deterministic downward time trend in presidential approval.

Table 4. Maximum Likelihood Estimates of d

	Parameter Estimates[a]	H_o: d = 1[b]	H_o: d = 0[b]
Macropartisanship	.787 (.104)	–2.05	7.57
Consumer Sentiment	.258 (.329)	–2.26	0.78
Presidential Approval	.261 (.297)	–2.49	0.88

[a]The standard errors of the estimates are shown in parentheses.
[b]These are the ML "*t*-ratios" for tests of the null hypothesis that d = 1 and d = 0.

Although these diagnostic tests can paint a broad picture of the degree of persistence in a time series, we can more accurately assess the degree of memory in a political process by obtaining point estimates of the order of integration and its standard error. Table 4 reports the ML estimates of d for each time series. These estimates were obtained by estimating ARFIMA(p,d,q) models with various combinations of up to three autoregressive and three moving average components for a total of 16 models. We then used the Schwarz Information Criterion (SIC) to select the best fitting model.[30]

As the results in Table 4 indicate, macropartisanship is fractionally integrated of order d = 0.787 since the null hypothesis that d = 1 is rejected ($t = -2.05$) as is the null hypothesis that d = 0 ($t = 7.57$). Thus, macropartisanship has a strong permanent component and is highly persistent. Despite the indications of a unit root in consumer sentiment provided by the DF and VR tests, our point estimates show that consumer sentiment is in fact stationary since the null hypothesis that d = 0 cannot be rejected ($t = 0.78$). The contrast between the diagnostic evidence and the point estimate of d occurs because consumer sentiment contains a large autoregressive component $(\rho_1 = 0.97)$, which shows up in the diagnostic tests as evidence of a unit root. Once we obtain separate estimates of the order of integration and the AR components, it becomes clear that the series is stationary but highly autoregressive. Finally, the point estimates for presidential approval suggest

[30]Mills (1992, 139) and Judge et al. (1985) note the advantages of using the SIC rather than the Akaike Information Criterion (AIC), each of which includes a penalty for adding parameters. In our empirical example, the SIC selected a (3,d,3) model for each of the three series. This is somewhat unusual since in other applications, the SIC selects more parsimonious models. Hamilton (1994, 449) and Granger (1980) suggest that ARMA models for fractionally integrated data will often contain an even larger number of AR and MA parameters.

that approval also is stationary since the null hypothesis of a unit root can be rejected ($t = -2.49$) while the null hypothesis of stationarity cannot ($t = 0.88$).[31]

Multivariate Models

Clearly, macropartisanship is fractionally integrated and contains a large permanent component, yet multivariate models of macropartisanship have failed to take this feature of the data into account. Ignoring the time series properties of macropartisanship will bias our inferences about both its degree of persistence and any short-run dynamics and will lead to model misspecfication when ARMA or ARIMA, rather than ARFIMA, transfer function models are estimated. To illustrate the problems of model misspecification and inference when time series are fractionally integrated, we estimate ARMA, ARFIMA, and ARIMA transfer function models of macropartisanship and examine their dynamic implications. The ARFIMA transfer function model we will use as a baseline is a stylized version of MacKuen, Erikson, and Stimson's (1989) first-order ARMA transfer function with an AR(1) noise model and is given as:

$$(1 - L)^{.79} y_t = \frac{\omega_{10}}{1 - \delta_{11}L} x_{1t} + \frac{\omega_{20}}{1 - \delta_{21}L} x_{2t} + \sum_{k=1}^{K} \beta_k z_{kt} + \frac{u_t}{1 - \delta_{31}L} \quad [3]$$

where y_t is macropartisanship, x_{1t} is consumer sentiment, x_{2t} is the political portion of a president's performance rating, u_t is a disturbance term, and the z_{kt} are control variables for the Vietnam and Gulf wars, the Watergate scandal, episodic historical events, and the first term of each president's administration.[32] Macropartisanship is fractionally differenced as indicated by the

[31]There has been considerable controversy over the time series properties of presidential approval (e.g., Beck 1993; Williams 1993; Ostrom and Smith 1993; Smith 1993; Alvarez and Katz 1996; DeBoef et al. 1996). Our results may provide insight into this empirical question.

We diagnose the original approval series here, while in the multivariate model, we use the "political" portion of approval and control for regime shifts with dummy variables for each administration. (See footnote 28 for evidence that "political" approval is stationary.) We use this approach because we want to make comparisons with MacKuen, Erikson, and Stimson (1989). A recent alternative approach to address changing administrations that was proposed by Green, Palmquist, and Schickler (1996) is to create a "party-centered" approval variable (see also DeBoef 1997): "This party-centering process can be accomplished simply by taking residuals from a regression in which approval (multiplied by –1 for Republican quarters) is regressed on a dummy variable scored 1 for Democratic quarters" (1996, 20).

[32]We measured the effects of the Vietnam War using the number of battle deaths for each quarter from the first quarter of 1965 through the fourth quarter of 1972. We measured the Watergate scandal using a dummy variable that took on the value of 1.0 from the second quarter of 1973 through the second quarter of 1974 and was zero otherwise. The effects of the Gulf War were measured with a dummy variable that took on the value of 1.0 in the first two quarters of 1991 and was

results in Table 4 and consumer sentiment and the political portion of approval are used in their levels form since the results in Table 4 (and fn. 28) indicate each series is stationary.

Following MacKuen, Erikson, and Stimson (1989), we multiply all of the independent variables (except the administration dummy variables) by a party variable that takes a value of 1 when the Democrats hold the presidency and a value of –1 otherwise. This ensures that an increase in a variable, such as the political portion of presidential approval, during a Republican administration causes the percentage of Democratic identifiers relative to the total to decline so that a single coefficient for the effects of an exogenous variable on macropartisanship can be estimated. The idea is to compare the differences between the correctly specified ARFIMA model with those that would be obtained by a researcher who naively used the macropartisanship data in levels or who automatically first differenced the macropartisanship series. We do so by changing the degree of differencing of the dependent variable so that we can specify ARMA, ARFIMA, and ARIMA transfer function models.

The results from estimating a first-order ARFIMA transfer function with an AR(1) noise component for macropartisanship appear in column 1 of Table 5. Using this model as a baseline, we can compare these estimates to those that would be obtained by analysts who either automatically first differenced their data (as for the ARIMA transfer function estimates in column 2 of Table 5) or who ignore the time series properties of the data and use the data in levels form (as in the ARMA transfer function estimates in column 3 of Table 5). Column 4 of Table 5 reports the estimates obtained by MacKuen, Erikson, and Stimson (1989), who specified an ARMA model and used data ending in the fourth quarter of 1987.

The results for the ARFIMA model show that consumer sentiment and the political portion of approval have statistically significant dynamic effects on fractionally differenced macropartisanship, but do not have statistically significant immediate effects. In addition, the Vietnam War variable is statistically significant. This can be compared to the results from the ARIMA model in which neither consumer sentiment nor the political portion of approval have any statistically significant effects (immediate or dynamic) on first differenced macropartisanship as well as to the results from the ARMA model in which consumer sentiment has a statistically significant immediate

zero otherwise. Our historical events series combines the event series used by MacKuen, Erikson, and Stimson (1989) with their variables for the Iran hostage crisis and the assassination attempt on Ronald Reagan. We also coded this variable a 1.0 from the fourth quarter of 1986 through the fourth quarter of 1988 to pick up the effects of the Iran-contra scandal and we gave this variable a value of –1.0 in the fourth quarter of 1990 to pick up the effects of the Bush budget battle.

Table 5. Estimates of Multivariate Transfer Functions

Model:	ARFIMA	ARIMA	ARMA	(MES 1989) ARMA
Dependent variable:	$(1-L)^{.79}y_t$	$(1-L)^1 y_t$	y_t	y_t
Constant	–.03	–.08	61.83	–2.78
	(.16)	(.14)	(1.49)	(.46)
consumer sentiment, ω_{10}	–.00	–.01	–.02	.10
	(.01)	(.01)	(.01)	(.01)
consumer sentiment, δ_{11}	1.03	–.77	.51	.84
	(.07)	(.60)	(.44)	(.02)
political approval, ω_{20}	–.01	.02	.07	.22
	(.01)	(.02)	(.03)	(.04)
political approval, δ_{21}	–1.02	–.73	.52	.35
	(.07)	(.78)	(.25)	(.09)
AR(1) noise component, δ_{31}	.02	–.20	.90	–.04
	(.09)	(.09)	(.04)	(.10)
Vietnam War	.35	–.00	.72	.56
	(.19)	(.18)	(.36)	
Watergate	.75	–.57	.11	–5.69
	(.81)	(.71)	(1.52)	
Historical events	.03	–.07	–.22	–1.38
	(.36)	(.34)	(.35)	
Gulf War	–1.26	1.16	.30	NA
	(1.27)	(1.22)	(1.29)	
DDE	—	—	—	4.71
JFK	–1.11	2.57	.38	—
LBJ	–1.73	2.11	.56	—
RMN	2.60	0.44	2.77	—
GRF	–1.77	1.58	.81	—
JEC	–.37	2.35	.65	17.91
RWR	1.19	–2.90	–.81	–5.26
GHB	2.10	–.29	.63	NA
Mean of dependent variable	.03	.05	61.39	—
SEE	1.74	1.76	1.71	1.83
DW	2.00	2.05	2.27	—
Q(35) (critical $\chi^2 = 49.52$)	55.87	55.62	46.69	—

effect but a statistically insignificant dynamic effect while the immediate and dynamic effects of the political portion of presidential approval are each statistically significant. The Vietnam War variable is also statistically significant in the ARMA specification.

With respect to each model's AR(1) noise component, we find that $\delta_{31} = .90$ and is statistically significant in the ARMA model; $\delta_{31} = -.20$ and is statistically significant in the ARIMA model;[33] and δ_{31} is not statistically different from zero in the ARFIMA model. The results from the ARMA model we estimated suggest that past levels of macropartisanship heavily influence its current value. When this long-run feature of the data is removed by fractional or first differencing, the effects of lagged values of the dependent variable are smaller or nonexistent. This is in contrast to the results from MacKuen, Erikson, and Stimson's ARMA model in which the AR(1) noise component was statistically insignificant. It is noteworthy that when the immediate effects of presidential approval are statistically significant as in the ARMA specification, the effect is smaller than that found in MacKuen, Erikson, and Stimson's original analysis. Furthermore, the statistically significant dynamic effects in both the ARFIMA and ARMA models are larger in absolute value than those reported by MacKuen, Erikson, and Stimson.[34]

More importantly, our empirical example shows that a researcher will come to very different conclusions about the effects of consumer sentiment, political approval, and the Vietnam War on macropartisanship depending upon the choice of model specification. An analyst using an ARMA specification would conclude that macropartisanship responds primarily to the political portion of approval. A researcher using an ARIMA specification would conclude that neither consumer sentiment nor political approval have an effect on macropartisanship. And an analyst using the correctly specified ARFIMA model would conclude that both consumer sentiment and political approval affect macropartisanship in the short run, but that those effects are dynamic rather than immediate. Clearly, the choice of model specification matters greatly in our attempts to explain the causes of movements in macropartisanship over time.

[33]The fact that $\delta_{31} = -0.20$ in the ARIMA model results from the overdifferencing of macropartisanship, which induces a moving average term in the series. When a MA(1) term is included in the transfer function model it is statistically significant while the AR(1) component no longer is, and the residuals from the model are then white noise. The critical Chi-square value for the Q statistic is 49.52, so the overdifferencing in the ARIMA model results in some serial correlation. There is also some serial correlation in the ARFIMA model, but after using a different noise model, the residuals of the ARFIMA model are white noise. We report our results using the MacKuen, Erikson, and Stimson (1989) first-order transfer function and noise model for comparative value. Since we do not have a lagged dependent variable in the ARIMA or ARFIMA models, the presence of serial correlation causes inefficiency but not bias.

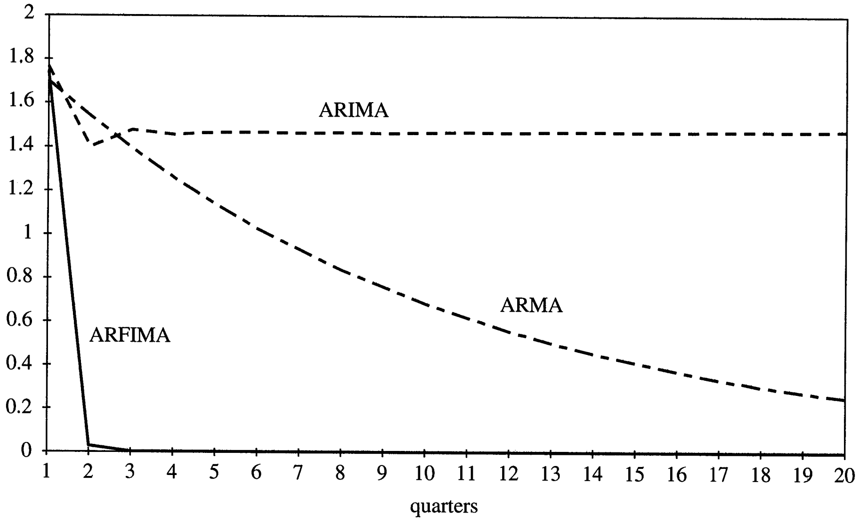
[34]There is a small degree of collinearity between the lagged effects of consumer sentiment and the political portion of approval in the ARFIMA model, and this inflates the *t*-ratios for those two variables somewhat. Additional testing indicates that it does not affect the standard errors or *t*-ratios of the other variables in the model.

Dynamic Implications

To illustrate further the differences in inference that result from different model specifications, we consider the dynamic implications of each of the three models. The decision to use the first differences, fractional differences, or levels of a series has important implications for the inferences an analyst reaches about the dynamic behavior of that process. For each of our three models, Figure 2 plots the dynamic response of macropartisanship to a one standard deviation exogenous shock. For the ARFIMA transfer function, the impact of the shock is an initial 1.74 percentage point increase; however, that effect decays to zero after just two quarters. For the ARIMA transfer function model, the impact of a shock to macropartisanship is similarly an initial 1.76 percentage point increase, but after some brief dampening of short-run dynamics in the first two quarters, the effect of the shock persists at 1.47 percentage points in every period. In stark contrast to the models using fractionally differenced and first differenced macropartisanship as the endogenous variable, the effect of a shock in the ARMA transfer function model is an initial 1.71 percentage points that dies out very gradually over time such that the series is still 0.25 percentage points above its equilibrium level after 20 quarters and is 0.03 percentage points above equilibrium after 40 quarters.

The evidence in Figure 2 fits with our expectations of the performance of these models. Since the first differenced and fractionally differenced endogenous variables in the ARIMA and ARFIMA models are stationary,

Figure 2. Transfer Function Dynamics



they will exhibit only short memory and tell us only about short-run dynamic behavior. However, because fractionally integrated series are mean reverting, the effect of a shock goes to zero in the ARFIMA model but persists at a new level in the ARIMA model. Finally, the dynamics from our ARMA model using the level of macropartisanship as the dependent variable comport well with Sowell's (1992b) argument that ARMA models cannot provide estimates of both long- and short-run dynamics when the series being modeled is fractionally integrated. In our application, the estimates are providing information about long-run effects.

Conclusions

The analyses in this paper have striking substantive and methodological implications. From a substantive viewpoint, it appears that MacKuen, Erikson, and Stimson (1989) were correct to argue that macropartisanship is related to factors such as consumer sentiment or the political portion of a president's approval rating, which have dynamic rather than immediate effects. Using a popular estimation approach of the time, their ARMA model specification and estimation techniques gave short shrift to the permanent component in macropartisanship. In contrast, our evidence of the existence of the short-run effects of shocks to macropartisanship as shown in the ARFIMA model should give pause to realignment and party system theorists while our evidence of a dominant long-run component in macropartisanship as shown in our estimate of d calls into question the arguments of students of macropartisanship. In short, we need theories and models of macropartisanship that can account for both its transitory and its permanent components.

From a methodological point of view, our analyses show how choices about the treatment of the time series properties of data and model specification heavily influence the inferences that an analyst reaches. In this empirical example, as shown in Figure 2, a researcher who used an ARMA specification with the data in levels would miss the transitory dynamics of macropartisanship, while an analyst who automatically first differenced the data and used an ARIMA specification would throw out important long-run information. Point estimates of d, the order of integration, however, can help researchers avoid such pitfalls. Our estimation of these three types of models highlights the importance of decisions regarding the memory of the series and choices about model specification. Strikingly different conclusions about the dynamic behavior of macropartisanship are reached based on choices about model specification and the treatment of the time series features of the data.[35]

[35]As researchers attempt to use tools for fractionally integrated time series in their own empirical work, they may run into instances in which the time series in question are a mix of different processes. For those cases, research continues. Practical solutions such as Granger's (1990, 12–134) "equation balancing" idea, in which analysts work to achieve a balance between the order of integration on the right- and left-hand sides of an equation, may also turn out to be useful in these cases.

As this analysis shows, techniques associated with fractionally integrated time series can shed light on important issues in political science such as the central question of change and stability in the United States party system. Because these tools allow us to avoid the binary choice between stationary and integrated alternatives and provide us with information about a continuum of dynamic behavior, they are likely to be useful in other political science contexts as well. Indeed, whenever the question is one of the degree of persistence in or of the dynamics of a political process, techniques for fractional integration provide an important new way for political scientists to test alternative hypotheses.

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APPENDIX A

In this appendix, we derive the autocorrelation function for a fractionally integrated series.

Theorem 1. The autocorrelation function for a fractionally integrated series is given by:

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \frac{(-d)!\,(k+d-1)!}{(d-1)!\,(k-d)!} \qquad \text{[A1]}$$

Proof:

To prove that this is true, divide Equation N1 shown in footnote 8 by Equation N2 in that footnote and invert and multiply. Then the ACF can be written as:

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \frac{(-1)^k(-d)!\,(-d)!}{(k-d)!\,(-k-d)!} = \frac{(-d)!}{(k-d)!} \times \frac{(-1)^k(-d)!}{(-k-d)!} \qquad \text{[A2]}$$

Since the first term of the last expression in Equation A2 contains the term (–d)!/(k – d)!, we can prove A1 simply by showing that:

$$\frac{(-1)^k(-d)!}{(-k-d)!} = \frac{(k+d-1)!}{(d-1)!} \qquad \text{[A3]}$$

To show that Equation A3 is true, we use the following facts:

$$-x! = \Gamma(1-x) \qquad \text{[A4a]}$$

$$x! = \Gamma(x+1) \qquad \text{[A4b]}$$

$$\Gamma(x)\Gamma(1-x) = \frac{\pi}{\sin \pi(x)} \Rightarrow \Gamma(1-x) = \frac{\pi}{\sin \pi(x)\Gamma(x)} \qquad \text{[A5]}$$

Use first fact A4a and then fact A5 and substitute into the numerator of A3 to obtain:

$$(-1)^k(-d)! = (-1)^k\Gamma(1-d) = \frac{(-1)^k\pi}{\Gamma(d)\sin \pi(d)} \qquad \text{[A6]}$$

And use first fact A4b and then fact A5 and substitute into the denominator of A3 to obtain:

$$(-k-d)! = \Gamma(-k-d+1) = \frac{\pi}{\sin \pi(-k-d)\Gamma(-k-d)} \qquad [A7]$$

Now divide Equation A6 by Equation A7 and invert and multiply to obtain:

$$\frac{(-1)^k(-d)!}{(-k-d)!} = \frac{(-1)^k \sin \pi(-k-d)\Gamma(-k-d)}{\Gamma(d)\sin \pi(d)} \qquad [A8]$$

Then substitute the following trigonometric identity:

$$(-1)^k \sin \pi(-k-x) = \sin \pi(-x) \qquad [A9]$$

into the numerator in Equation A8 and re-express to obtain:

$$\frac{(-1)^k(-d)!}{(-k-d)!} = \frac{\sin \pi(-d)\Gamma(-k-d)}{\sin \pi(d)\Gamma(d)} = \frac{(-1)\sin \pi(d)\Gamma(-k-d)}{\sin \pi(d)\Gamma(d)} \qquad [A10]$$

Then cancel like terms, apply fact A4a, and use the fact that $(x-1)! = \Gamma(x)$ to obtain:

$$\frac{(-1)^k(-d)!}{(-k-d)!} = \frac{(-1)\Gamma(-k-d)}{\Gamma(d)} = \frac{(-1)1-(-k-d)!}{(d-1)!} = \frac{(k+d-1)!}{(d-1)!} \qquad [A11]$$

This completes the proof.

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