Statistics Corner

"Interpreting Interaction Terms with Continuous Variables in Ordinary Least Squares Regression."

By Kevin Sweeney

In our effort to make Lab Notes more interesting and informative to our readers, the Lab is pleased to add a new feature. In each issue one page will be dedicated to a vexing methodological topic. Our hope is that lots of folks in the department will contribute to this page on a regular basis. We encourage all to forward potential topics to Sweeney. You never know, maybe we'll write about it. Since this was my brilliant idea, I get to go first; and because of the introduction I have less than a page to work with – Yikes!

The world is not nearly as linear and additive as it seems from looking at the statistical models that get published in political science journals. A familiar looking regression model, like (1),

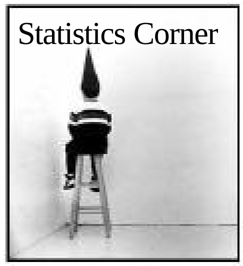
$$Y = \hat{b}_0 + \hat{b}_1 X_1 + \hat{b}_2 X_2 + e \qquad (1)$$

assumes the effect of an independent variable X_1 on the dependent variable Y is always the same, regardless of the values of the other independent variables. In the complex world of social science it often makes sense to allow for the possibility that the effects of one independent variable *vary* according to the value of another. If this were the case, we would estimate a model like (2) where the interaction between $X_1 X_2$ is

$$Y = \hat{b}_0 + \hat{b}_1 X_1 + \hat{b}_2 X_2 + \hat{b}_3 X_1 X_2 + e \qquad (2)$$

added to the model. The remainder of this page will consider this type of interaction for OLS, where both X_1 and X_2 are continuous variables. You should know that interpreting interaction terms in the case of binary independent or limited dependent variables is somewhat different.

If you specified an equation like (2) it would be ***incorrect*** to interpret either $\hat{b}_1$ or $\hat{b}_2$ as the marginal effect on Y for a one unit change in



X_1 or X_2 respectively, unless you were only interested in the cases where X_1 and $X_2 = 0$. Your estimate of b_1 (and its standard error) is only the familiar marginal effect for $X_2 = 0$, and your estimate of b_2 (and its standard error) is only for $X_1 = 0$. Given that 0 may fall outside the observable range of one, or both, of these variables – this is not very useful.

The correct formulas for conditional interpretation of coefficients (and their standard errors) across the range of both covariates in the interaction are:

$$\hat{b}_1 \quad at \quad X_2 = \hat{b}_1 + \hat{b}_3 X_2$$

$$\hat{b}_2 \quad at \quad X_1 = \hat{b}_2 + \hat{b}_3 X_1$$

$$SE(\hat{b}_1 at X_2) = \left[\text{var}(\hat{b}_1) + X_2^2 \text{ var}(\hat{b}_3) + 2 X_2 \text{ cov}(\hat{b}_1, \hat{b}_3)\right]^{\frac{1}{2}}$$

$$SE(\hat{b}_2 at X_1) = \left[\text{var}(\hat{b}_2) + X_1^2 \text{ var}(\hat{b}_3) + 2 X_1 \text{ cov}(\hat{b}_2, \hat{b}_3)\right]^{\frac{1}{2}}$$

where var is variance and cov is covariance. This information is readily available post-estimation from most statistical packages - simply retrieve the variance-covariance matrix.

For an economical presentation of this rich information, it may be useful to graph each coefficient across the range of the other variable. If you do this you will notice that coefficients may switch signs and/or lose and regain statistical significance across the range of the interaction, and you will notice (if you haven't already) that your estimated coefficients for each variable are only good when the other variable in the interaction is zero.

Good References:

Friedrich, Robert J. 1982. "In Defense of Multiplicative Terms in Multiple Regression Equations." *AJPS* 26:797-834.

Jaccard, James et al. 2003. *Interaction Effects in Multiple Regression Sage* #72.