Probability Problems: Exam-Type Examples

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1) How big are farms? A scholar chose American farms at random and measured their sizes in acres. Here are the probabilities that the one farm chosen falls in several acreage categories:

Acres	<10	10-49	50-99	100-179	180-499	500-999	1000-1999	2000
Probability	0.09	0.20	0.15	0.16	0.22	0.09	0.05	0.04

Let A be the probability that the farm is less than 50 acres in size (i.e. P(A) < 50) and let B be the probability that the farm is 500 acres or more (i.e. P(B) 500).

a) Find P(A) and P(B).

b) Find the probability that A does not occur (i.e. P (~A)).

c) Find the probability that A or B occurs (i.e. P (A or B)).

→ Hint: Keep in mind that these events are disjoint—there is not overlap between them…

2) A roulette wheel has 28 slots, numbered 0, 00, and 1 to 36. The slots 0 and 00 are colored green, 18 of the others are red, and 18 are black. The dealer spins the wheel and at the same time a small ball along the wheel rolls in the opposite direction. The wheel is carefully balanced so that the ball is equally likely to land in any slot when the wheel slows. Gamblers can bet on various combinations of numbers and colors.

   a) What is the probability of any one of the 38 possible outcomes?

   b) If you bet on "red," you win if the ball lands in a red slot. What is the probability of winning?

   c) The slot numbers are laid out on a board on which gamblers place their bets. One column of numbers on the board contains all multiples of 3 (i.e. 3, 6, 9, 12, 15, …, 36). You place a "column bet" if any of these multiples of three comes up. What is your probability of winning?

3) In each of the following situations, state whether or not the given assignment of probabilities to individual outcomes is legitimate, that is, satisfies the rules of probability. If not, give specific reasons for your answer.

   a) When a coin is spun, P(H) = 0.55 and P(T) = 0.45.
   b) When two coins are tossed, P(HH) = 0.4, P(HT) = 0.4, P(TH) = 0.4, and P(TT) = 0.4

c) Plain M&M's have not always had the mixtures of colors they have today. In the past, there were no red candies and no blue candies. The color "tan" had the probability 0.10 and the four other colors had the probabilities 0.3 (brown), 0.2 (yellow), 0.1 (green), and 0.1 (orange). If these are all of the colors in a bag of Plain M&M's, is this a legitimate probability outcome? Why or why not?

4. You have a torn tendon and are facing surgery to repair it. The surgeon explains the risks to you: infection occurs in 3% of such operations, the repair fails in 14%, and both infection and failure occur together in 1%. What percent of these operations succeed and are free from infection?

5. A poker player holds a flush when all five cards in the hand belong to the same suit. We will find the probability of a flush when five cards are dealt. Remember that a deck contains 52 cards, 13 of each suit, and that when the deck is well shuffled, each card dealt is equally likely to be any of those that remain in the deck.

a) We will concentrate on spades. What is the probability that the first card dealt is a spade? What is the conditional probability that the second card dealt is a spade, given that the first is a spade?

b) Continue to count the remaining cards to find the conditional probabilities of a spade on the third, the fourth, and the fifth card, given in each case that all previous cards are spades.

c) The probability of being dealt five spades is the product (i.e. multiply) of the five probabilities you have found. Why? What is this probability?

d) The probability of being dealt five hearts or five diamonds or five clubs is the same as the probability of being dealt five spades. What is the probability of being dealt a flush?

Answers for Probability Problems

1. a) P(A) = P(X < 50) = 0.09 + 0.20 = 0.29
P(B) = P(X   500) = 0.09 + 0.05 + 0.04 = 0.18

b) P(~A) = 1 – P(A) = 1 – 0.29 = 0.71 (i.e. the probability that the farm is over 50 acres in size).

c) P(A or B) = P(A) + P(B) = 0.29 + 0.18 = 0.47

2. a) P(1 of 38) = 1 / 38 = 0.026

b)

$$P(\text{Red}) = \frac{18 \text{ (the \# of red outcomes)}}{38 \text{ (the total number of possible outcomes)}}$$

$$= 0.474$$

c) There are 12 multiples of 3 between 1 and 38 (3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, and 36). To find the probability of winning on our column bet, we just multiply 12 times the probability of one of the events occurring:

$$P(\text{multiple of 3}) = 12 * \frac{1}{38} = 0.316$$

3. a) This is legitimate because the probability of one event, P(H), plus the probability of the other, P(T), gives us a total probability of 1.

P(H) + P(T) = 0.55 + 0.45 = 1.0

b) This is not legitimate because the sum of the probabilities is greater than 1:

P(HH) + P(HT) + P(TH) + P(TT) = 0.4 + 0.4 + 0.4 + 0.4 = 1.6 which is > 1

c) This is not legitimate because the probabilities all add up to less than one even though we’ve exhausted all of the possible colors in the bag:

P(tan) + P(brown) + P(yellow) + P(green) + P(orange)

= 0.1 + 0.3 + 0.2 + 0.1 + 0.1 = 0.8 which is < 1

4. P(infection) = 0.03
P(failed repair) = 0.14
P(failure & infection) = 0.01

Using our General Addition Rule we find:

P(A or B) = P(A) + P(B) – P(A and B)

= P(infection) + P(failed repair) – P(infection & failed repair)

= 0.03 + 0.14 – 0.01

= 0.16

To find the probability that the repair is successful and free from infection, we just need to subtract the probability of each possible event occurring from 1:

P (not fail and not infection) = P(succeed & no infection) = 1 – P(failed or infection)

= 1 – 0.16 = 0.84

5. a. P(1st card spade) = $\frac{13}{52}$ = 0.25

P(2nd card spade | 1st card spade) = $\frac{12}{51}$ = 0.2353

b. P(3rd card spade | 1st and 2nd spades) = $\frac{11}{50}$ = 0.22

P(4th card spade | 1st - 3rd spades) = $\frac{10}{49}$ = 0.2041

P(5th card spade | 1st – 4th spades) = $\frac{9}{48}$ = 0.1875

c. P(spade flush) is the product of the above numbers such that:

P(spade flush) = 0.2353*0.22*0.2041*0.1875 = 0.000495

d. Since there are four possible suits where a flush can occur, the probability of getting a flush is 4 times the probability in (c):

= 4 * 0.00495 = 0.001981