

## Binomial Distribution Examples

(where you see **n** make sure you put big parentheses around them)

**k**

The formula:  $P(X = k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$

where

- $n =$  the number of observations we have in our sample
- $k =$  the number of “successes” or “failures” we’re interested in (this depends on how the question is asked)
- $p =$  the probability of being a success (or failure, depending on how the question is asked)
- $n! =$  a factorial ( $n*(n-1)*(n-2)*\dots*3*2*1$ )

1. Among employed women, 25% have never been married. Select 10 employed women at random.
  - a) The number in your sample who have never been married has a binomial distribution. What are  $n$  and  $p$ ?

To do this, we first need to turn to our formula (and the “where” definition statements). We see that our sample of women = 10, so:

$$n = 10$$

The probability of being a “success” (i.e. of being married) is 0.25 (25% / 100), so:

$$p = 0.25$$

And the number of “successes” we’re looking for, i.e. the number of women who are married, is 2. So:

$$k = 2$$

- b) What is the probability that exactly 2 of the 10 women in your sample have never been married?

Let’s put our formula to use:

From above:

$$p = 0.25 \quad n = 10 \quad k = 2$$

So:

$$P(X = k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

is,

$$\begin{aligned} P(X = 2) &= \frac{10!}{2!(10-2)!} 0.25^2 (0.75)^8 \\ &= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} (0.0625) \cdot (0.10013) \\ &= \frac{10 \cdot 9 \cdot 8}{2 \cdot 1} (0.006257) \\ &= 45 \cdot (0.006257) \\ &= 0.2816 \end{aligned}$$

c) What is the probability that 2 or fewer have never been married?

Same theory here as above, only this time we have to take into account the probability of two, one, or zero women having never been married:

$$\begin{aligned} P(X \leq 2) &= \frac{10!}{2!(10-2)!} (0.25)^2 (0.75)^8 + \frac{10!}{1!(10-1)!} (0.25)^1 (0.75)^9 + \frac{10!}{0!(10-0)!} (0.25)^0 (0.75)^{10} \\ P(X \leq 2) &= \frac{10!}{2} (0.006258) + \frac{10!}{1} 0.018771 + \frac{10!}{0} 0.0056314 \\ &= 45 \cdot (0.006258) + 10 \cdot (0.018771) + 1 \cdot (0.0056314) \\ &= 0.5256 \end{aligned}$$

So the probability of finding 2 or fewer women who have never been married is 0.5256.

\*\*For every permutation, make sure that you write out all of the digits and then cancel them out where appropriate. Questions about this? See Bill or your notes...

Let's look at one more example:

In a test for ESP, a subject is told that cards the experimenter can see but he/she cannot contain either a star, a circle, a wave, or a square. As the experimenter looks at each of 20 cards in turn, the subject names the shape on the card. A subject who is just guessing has probability 0.25 of guessing correctly on each card.

- a) The count of correct guesses in 20 cards has a binomial distribution. What are  $n$  and  $p$ ?

We know that there are twenty observations in our sample, so

$$n = 20$$

We also know that the probability of guessing correctly is 0.25, so

$$p = 0.25$$

- b) What is the probability of exactly 5 correct guesses?

We again use the tried and true formula:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where, in this case we want to know what the probability of 5 correct guesses is. Thus, we know that we're trying to figure out the probability of 5 "successes". So,

$$k = 5$$

Thus:

$$\begin{aligned} P(X = 5) &= \binom{20}{5} (0.25)^5 (0.75)^{15} \\ &= \frac{20!}{5!15!} (0.000977) * (0.013363) \\ &= \frac{(20*19*18*17*16)}{(5*4*3*2*1)} * (0.0000130503) \\ &= (15504) * (0.000013053) = 0.202 \end{aligned}$$