ERROR COMPONENTS MODELS

1. Model and Notation

Consider a linear regression equation of the form:

$$y_{it} = z_i'\alpha + x_{it}'\beta + u_{it} = w_{it}'\delta + u_{it} \tag{1.1}$$

where $i = 1, \ldots, N$ indexes individuals (or states, counties, etc.) and $t = 1, \ldots, T$ indexes time periods. The *error components* model specifies that the error u_{it} is the sum of component errors. Terminology varies slightly, but usually a model in which there are two components is called "one-way" and a model in which there are three components is called "two-way." Although those labels might seem perverse taken in isolation, they are exactly anaologous to the ANOVA terminology from week 1. For instance, a "one-way ANOVA" model $y_{ij} = \mu + \alpha_i + e_{ij}$ had both the α terms representing the "one-way" classification introduced by some factor and an additional random error, e_{ij}. So one one-way error components model posits:

$$u_{it} = \mu_i + \nu_{it} \tag{1.2}$$

and another is:

$$u_{it} = \eta_t + \nu_{it} \tag{1.3}$$

We will focus on (1.2) first. The presence of the individual specific effect μ_i allows for intercorrelation among the T error terms, $u_{i1}, \ldots, u_{iT}$, for individual i.

Various assumptions can be made about the regressors and error components, and they determine the form of the error covariance structure. The $k \times 1$ vector of time-varying regressors x_{it} and the $p \times 1$ vector of time-invariant regressors, z_i are assumed to be fixed. The $(k+p) \times 1$ vectors $w_{it} = (z_i', x_{it}')'$ and $\delta = (\alpha', \beta')'$ are partioned conformably.

The individual effect will be treated as a random variable with mean zero:

$$E\mu_i = 0 \text{ for } i = 1, \ldots, N \tag{1.4}$$

We will also (for now) assume homoscedasticity and non-autocorrelation:

$$E\mu_i\mu_j = \begin{cases} \sigma_\mu^2, & \text{for } i = j \\ 0, & \text{otherwise.} \end{cases} \tag{1.5}$$

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Similarly, ν_{it} is assumed to have mean zero:

$$E\nu_{it} = 0 \text{ for } i = 1, \ldots, N \text{ and } t = 1, \ldots, T, \tag{1.6}$$

and to be homoscedastic and nonautocorrelated:

$$E\nu_{is}\nu_{jt} = \begin{cases} \sigma_\mu^2, & \text{for } i = j \text{ and } s = t \\ 0, & \text{otherwise.} \end{cases} \tag{1.7}$$

Finally, the error components are assumed to be uncorrelated with one another:

$$E\mu_i\nu_{jt} = 0 \text{ for } j = 1, \ldots, N \text{ and } t = 1, \ldots, T. \tag{1.8}$$

Note that the individual-specific effect for individual i is uncorrelated with both his (or her, or its) own residual error ν_{it} and that for each other individual.

The T observations for person i can be written in matrix notation as:

$$y_i = (\mathbf{1}_T \otimes z_i')\alpha + X_i\beta + u_i = W_i\delta + u_i \tag{1.9}$$

where:

$$\underset{T\times 1}{y_i} = \begin{bmatrix} y_{i1} \\ \vdots \\ y_{iT} \end{bmatrix} \quad \underset{T\times k}{X_i} = \begin{bmatrix} x_{i1}' \\ \vdots \\ x_{iT}' \end{bmatrix} \quad \underset{T\times 1}{u_i} = \begin{bmatrix} u_{i1} \\ \vdots \\ u_{iT} \end{bmatrix}$$

and $W_i = (\mathbf{1}_T \otimes z_i', X_i)$. Similarly, the $T \times 1$ disturbance vector for person i can be written as:

$$u_i = \mu_i \mathbf{1}_T + \nu_i \tag{1.10}$$

where $\nu_i = (\nu_{i1}, \ldots, \nu_{iT})'$.

Next, we can stack the N vector equations (1.9) to obtain:

$$y = Z\alpha + X\beta + u = W\delta + u \tag{1.11}$$

where:

$$\underset{NT\times 1}{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} \quad \underset{NT\times p}{Z} = \begin{bmatrix} \mathbf{1}_T \otimes z_1' \\ \vdots \\ \mathbf{1}_T \otimes z_N' \end{bmatrix} \quad \underset{NT\times k}{X} = \begin{bmatrix} X_1 \\ \vdots \\ X_N \end{bmatrix} \quad \underset{NT\times 1}{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix}$$

and $W = (Z, X)$. It will be assumed that W has rank $k + p$. The analogue of equation (1.10) combining all NT observations is:

$$u = \mu \otimes \mathbf{1}_T + \nu \tag{1.12}$$

where

$$\underset{N\times 1}{\mu} = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_N \end{bmatrix} \quad \text{and} \quad \underset{NT\times 1}{\nu} = \begin{bmatrix} \nu_1 \\ \vdots \\ \nu_{NT} \end{bmatrix}$$

These are fairly strong assumptions. There are no time effects and the unobservable individual effects are additive. Most important, assuming x_{it} fixed and $E\mu_i = 0$ implies that the unobservable effects are uncorrelated with the regressors. We will eventually consider some tests of this assumption .

2. Error Covariance Structure

In this section, we will investigate the covariance structure of the error components model. Let $\Omega = Euu'$ denote the covariance matrix of u (which is of order NT). Expressions for Ω, Ω^{-1}, and $\Omega^{-\frac{1}{2}}$ will be derived.

First, consider the individual-specific effects μ_i:

$$E\mu\mu' = \begin{bmatrix} E\mu_1^2 & \dots & E\mu_1\mu_N \\ \vdots & \ddots & \vdots \\ E\mu_N\mu_1 & \dots & E\mu_N^2 \end{bmatrix} = \sigma_\mu^2 I_N \tag{2.1}$$

using assumption (1.5). Similarly, assumption (1.6) implies that:

$$E\nu_i\nu_j' = \begin{cases} \sigma_\nu^2 I_T & \text{if } i = j \\ \mathbf{0}_{T\times T} & \text{otherwise} \end{cases}$$

so that:

$$E\nu\nu' = \begin{bmatrix} \sigma_\nu^2 I_T & \dots & \mathbf{0}_{T\times T} \\ \vdots & \ddots & \vdots \\ \mathbf{0}_{T\times T} & \dots & \sigma_\nu^2 I_T \end{bmatrix} = \sigma_\nu^2 I_{NT} \tag{2.2}$$

Using (2.1) and (2.2), we obtain from equation (1.12):

$$\begin{aligned} \Omega &= E(\mu \otimes \mathbf{1}_T + \nu)(\mu \otimes \mathbf{1}_T + \nu)' \\ &= E(\mu\mu') \otimes \mathbf{1}_T\mathbf{1}_T' + E\nu\nu' \\ &= \sigma_\mu^2 I_N \otimes J_T + \sigma_\nu^2 I_{NT} \end{aligned}$$

since μ and ν are uncorrelated by assumption (1.8). Thus Ω has a block diagonal form:

$$\Omega = I_N \otimes \Sigma \tag{2.3}$$

where:

$$\Sigma = \sigma_\nu^2 I_T + \sigma_\mu^2 J_T.$$

Σ is the covariance matrix of u_i. Each diagonal element of Σ equals $\sigma_\nu^2+\sigma_\mu^2$, the variance of u_{it}. The (s,t)th off-diagonal element of Σ is equal to σ_μ^2, the covariance between u_{is} and u_{it}. Thus the form of Σ implies that the elements of u_i are equicorrelated:

$$\rho = corr(u_{is}, u_{it}) = \frac{\sigma_\mu^2}{\sigma_\nu^2 + \sigma_\mu^2} \quad (s \neq t)$$

Note that this correlation must be non-negative.

To compute Ω^{-1} note that:

$$\Omega^{-1} = (I_N \otimes \Sigma)^{-1} = I_N \otimes \Sigma^{-1} \tag{2.4}$$

so it is necessary to compute Σ^{-1}.

Proposition 2.1. $\Sigma^{-1} = \frac{1}{\sigma_\nu^2}(I_T - \frac{\sigma_\mu^2}{\sigma_\nu^2+T\sigma_\mu^2}J_T)$.

Proof. Let $a = \sigma_\mu^2/(\sigma_\nu^2 + T\sigma_\mu^2)$. Then:

$$\begin{aligned}
\frac{1}{\sigma_\nu^2}(I_T - aJ_T)\Sigma &= \frac{1}{\sigma_\nu^2}(I_T - aJ_T)(\sigma_\nu^2 I_T + \sigma_\mu^2 J_T) \\
&= I_T + \frac{1}{\sigma_\nu^2}(\sigma_\mu^2 J_T - a\sigma_\nu^2 J_T - a\sigma_\mu^2 J_T J_T) \\
&= I_T + \frac{\sigma_\mu^2 - a(\sigma_\nu^2 + T\sigma_\mu^2)}{\sigma_\nu^2} J_T \\
&= I_T. \quad \square
\end{aligned}$$

In the following section, it will be useful to factor Ω^{-1} as the square of another matrix. For any symmetric positive definite matrix A, there exists a unique matrix $A^{\frac{1}{2}}$ such that $A = A^{\frac{1}{2}}A^{\frac{1}{2}}$. Since Ω is positive definite, Ω^{-1} is positive definite as well, so there exists a unique matrix $\Omega^{-\frac{1}{2}}$ such that $\Omega^{-\frac{1}{2}}\Omega^{-\frac{1}{2}} = \Omega^{-1}$.

Proposition 2.2. $\Omega^{-\frac{1}{2}} = I_N \otimes \Sigma^{-\frac{1}{2}}$ where:

$$\Sigma^{-\frac{1}{2}} = \frac{1}{\sigma_\nu}\left(I_T - \frac{1-\sqrt{\theta}}{T}J_T\right)$$

and $\theta = \sigma_\nu^2/(\sigma_\nu^2+T\sigma_\mu^2)$. Caution: this θ is not quite the same as Baltagi's θ (introduced on page 15). Call Baltagi's term θ_B. Then the θ above is related by:

$$\theta_B = 1 - \sqrt{\theta}$$

Proof.

$$\left(I_T - \frac{1-\sqrt{\theta}}{T}J_T\right)^2 = I_T - \frac{2(1-\sqrt{\theta})}{T}J_T + \frac{1-2\sqrt{\theta}+\theta}{T^2}J_TJ_T$$
$$= I_T - \frac{1-\theta}{T}J_T$$
$$= I_T - \frac{\sigma_\mu^2}{\sigma_\nu^2 + T\sigma_\mu^2}J_T$$

so:

$$\left(\frac{1}{\sigma_\nu}\left(I_T - \frac{1-\sqrt{\theta}}{T}J_T\right)\right)^2 = \frac{1}{\sigma_\nu^2}\left(I_T - \frac{\sigma_\mu^2}{\sigma_\nu^2 + T\sigma_\mu^2}J_T\right) = \Sigma^{-1}$$

by Proposition 2.1, thus giving $\Sigma^{-\frac{1}{2}}$. To obtain $\Omega^{-\frac{1}{2}}$, note that:

$$(I_N \otimes \Sigma^{-\frac{1}{2}})^2 = I_N \otimes \Sigma^{-\frac{1}{2}}\Sigma^{-\frac{1}{2}} = I_N \otimes \Sigma^{-1}. \quad \square$$

3. Generalized Least Squares Estimation

Since the covariance matrix Ω of u is non-spherical (i.e. not equal to a scalar multiplied by an identity matrix), ordinary least squares is inefficient (though still unbiased). We proceed to develop the Generalized Least Squares (GLS) estimator of δ by treating Ω as known. The subsequent problem will then be the estimation of Ω.

Prior notes reviewed the *within groups* estimator $\hat{\beta}_W$ of β:

$$\hat{\beta}_W = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{y}$$

where $\tilde{X}$ and $\tilde{y}$ consist of the values of x_{it} and y_{it} deviated from their group means, e.g.

$$\tilde{x}_{it} = x_{it} - \bar{x}_{i\cdot} \quad \text{and } \bar{x}_{i\cdot} = \frac{1}{T}\sum_{t=1}^{T} x_{it} \quad \text{etc.}$$

We motivated the terminology "within group" by starting from the perspective of analyis of variance and the sum of squares decomposition, $TSS = WSS + BSS$. There is also, of course, *between groups* estimator, $\hat{\delta}_B = (\hat{\alpha}'_B, \hat{\beta}'_B)'$ which is obtained by regressing $y_{i\cdot}$ on z_i and $x_{i\cdot}$:

$$\hat{\delta}_B = \begin{pmatrix}\hat{\alpha}_B \\ \hat{\beta}_B\end{pmatrix} = \begin{pmatrix}\sum_{i=1}^N z_i z_i' & \sum_{i=1}^N z_i \bar{x}_{i\cdot}' \\ \sum_{i=1}^N \bar{x}_{i\cdot} z_i' & \sum_{i=1}^N \bar{x}_{i\cdot}\bar{x}_{i\cdot}'\end{pmatrix}^{-1} \begin{pmatrix}\sum_{i=1}^N z_i \bar{y}_{i\cdot} \\ \sum_{i=1}^N \bar{x}_{i\cdot}\bar{y}_{i\cdot}\end{pmatrix}$$

(Although we didn't develop the estimator in the theoretical notes last time, I replicated the between estimates for the Grunfeld example both with a canned Stata routine and by estimating the regression mentioned above (pages 46-49).)

Recall how we defined the symmetric idempotent matrices $\bar{J}_T$ and C_T:

$$\bar{J}_T = \frac{1}{T}\mathbf{1}_T\mathbf{1}_T' \quad \text{and } C_T = I_T - \bar{J}_T$$

Then:

$$\bar{J}_T X_i = \mathbf{1}_T \bar{x}_{i\cdot}' \quad \text{and } C_T X_i = \tilde{X}_i$$

and similarly for y_i and $\tilde{y}_i$. Now we define the within and between sums of squares (with a new double-subscript notation):

$$W_{xx} = \sum_{i=1}^{N} \tilde{X}_i'\tilde{X}_i = \sum_{i=1}^{N} X_i' C_T X_i \tag{3.1}$$

$$B_{xx} = \sum_{i=1}^{N} \bar{x}_{i\cdot}\bar{x}_{i\cdot}' = \frac{1}{T}\sum_{i=1}^{N} X_i'\bar{J}_T X_i \tag{3.2}$$

$$B_{zz} = \sum_{i=1}^{N} z_i z_i' = \frac{1}{T}\sum_{i=1}^{N} Z_i'\bar{J}_T Z_i \tag{3.2}$$

(Note that the matrix $\bar{J}_T$ is redundant in the equation for B_{zz} since $\bar{J}_T Z_i = Z_i$; however, we wish to emphasize the parallel structure, and the expression will reappear.) In this notation, the within-groups estimator is:

$$\hat{\beta}_W = W_{xx}^{-1} W_{xy} \tag{3.4}$$

where

$$W_{xy} = \sum_{i=1}^{N} \tilde{X}_i'\tilde{y}_i = \sum_{i=1}^{N} X_i' C_T \bar{y}_i. \tag{3.5}$$

The between-groups estimator is:

$$\hat{\delta}_B = \begin{pmatrix} B_{zz} & B_{zx} \\ B_{xz} & B_{xx} \end{pmatrix}^{-1} \begin{pmatrix} B_{zy} \\ B_{xy} \end{pmatrix} \tag{3.6}$$

where:

$$B_{xz} = \sum_{i=1}^{N} \bar{x}_{i\cdot} z_i' = \frac{1}{T}\sum_{i=1}^{N} X_i'\bar{J}_T Z_i \tag{3.7}$$

$$B_{xy} = \sum_{i=1}^{N} \bar{x}_{i\cdot}\bar{y}_{i\cdot} = \frac{1}{T}\sum_{i=1}^{N} X_i'\bar{J}_T y_i \tag{3.8}$$

$$B_{zy} = \sum_{i=1}^{N} z_i \hat{y}_{i\cdot} = \frac{1}{T}\sum_{i=1}^{N} Z_i'\bar{J}_T y_i \tag{3.9}$$

Using the results on two-step least squares from the first lecture notes (or see the partitioned inverse formula, e.g. in Theil *Principles of Econometrics*, pp. 17-18), it follows that:

$$\hat{\beta}_B = (B_{xx} - B_{xz}B_{zz}^{-1}B_{zx})^{-1}(B_{xy} - B_{xz}B_{zz}^{-1}B_{zy}) \tag{3.10}$$

$$\hat{\alpha}_B = B_{zz}^{-1}B_{zy} - B_{zz}^{-1}B_{zx}\hat{\beta}_B \tag{3.11}$$

The key results are (3.4), (3.10), and (3.11).

With these results we can develop the GLS estimation technique. Proposition (3.1) indicates that the GLS estimator of β is a weighted average of the within groups and between groups estimators.

Proposition 3.2. $\hat{\alpha}_G = \hat{\alpha}_B$ and $\hat{\beta}_G = \Delta\hat{\beta}_W + (I_k - \Delta)\hat{\beta}_B$ where:

$$\Delta = (W_{xx} + \theta(B_{xx} - B_{xz}B_{zz}^{-1}B_{zx}))^{-1}W_{xx}.$$

Proof. The usual formula for the GLS estimator is:

$$\hat{\delta}_G = (W'\Omega^{-1}W)^{-1}W'\Omega^{-1}y$$

Note that:

$$W'\Omega^{-1}W = \begin{bmatrix} W_1 \\ \vdots \\ W_N \end{bmatrix}' \begin{bmatrix} \Sigma^{-1} & \dots & (0) \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \Sigma^{-1} \end{bmatrix} \begin{bmatrix} W_1 \\ \vdots \\ W_N \end{bmatrix} = \sum_{i=1}^{N} W_i \Sigma^{-1} W_i \tag{3.12}$$

and, similarly,

$$W'\Omega^{-1}y = \sum_{i=1}^{N} W_i'\Sigma^{-1}y_i$$

so (3.12) becomes:

$$\hat{\delta}_G = \left(\sum_{i=1}^{N} W_i'\Sigma^{-1}W_i\right)^{-1} \left(\sum_{i=1}^{N} W_i'\Sigma^{-1}y_i\right)^{-1} \tag{3.13}$$

From Proposition 2.1:

$$\begin{aligned} \sigma_\nu^2\Sigma^{-1} &= \left(I_T - \frac{1}{T}J_T\right) + \left(\frac{1}{T} - \frac{\sigma_\mu^2}{\sigma_\nu^2 + T\sigma_\mu^2}\right) J_T \\ &= C_T + \frac{\theta}{T}\bar{J}_T \end{aligned} \tag{3.14}$$

so, in turn, (3.13) becomes:

$$\hat{\delta}_G = \left(\sum_{i=1}^{N} W_i' C_T W_i + \frac{\theta}{T} \sum_{i=1}^{N} W_i' \bar{J}_T W_i \right)^{-1} \left(\sum_{i=1}^{N} W_i' C_T y_i + \frac{\theta}{T} \sum_{i=1}^{N} W_i' \bar{J}_T W_i \right)^{-1} \tag{3.15}$$

Further simplification follows from:

$$\sum_{i=1}^{N} W_i' C_T W_i = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & W_{xx} \end{pmatrix}$$

$$\frac{1}{T} \sum_{i=1}^{N} W_i' \bar{J}_T W_i = \begin{pmatrix} B_{zz} & B_{zx} \\ B_{xz} & B_{xx} \end{pmatrix}$$

$$\sum_{i=1}^{N} W_i' C_T y_i = \begin{pmatrix} \mathbf{0} \\ W_{xy} \end{pmatrix}$$

$$\frac{1}{T} \sum_{i=1}^{N} W_i' \bar{J}_T y_i = \begin{pmatrix} B_{zy} \\ B_{xy} \end{pmatrix}$$

Substituting the equations above into (3.15) yields:

$$\begin{pmatrix} \hat{\alpha}_G \\ \hat{\beta}_G \end{pmatrix} = \begin{pmatrix} \theta B_{zz} & \theta B_{zx} \\ \theta B_{xz} & W_{xx} + \theta B_{xx} \end{pmatrix}^{-1} \begin{pmatrix} \theta B_{zy} \\ W_{xy} + \theta B_{xy} \end{pmatrix} \tag{3.16}$$

An application of the partitioned inverse formula yields:

$$\begin{pmatrix} \theta B_{zz} & \theta B_{zx} \\ \theta B_{xz} & W_{xx} + \theta B_{xx} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{\theta} B_{zz}^{-1} + B_{zz}^{-1} B_{zx} C^{-1} B_{xz} B_{zz}^{-1} & -B_{zz}^{-1} C^{-1} \\ -C^{-1} B_{xz} B_{zz}^{-1} & C^{-1} \end{pmatrix} \tag{3.17}$$

where $C = W_{xx} - \theta(B_{xx} - B_{xz} B_{zz}^{-1} B_{zx})$. Substituting (3.17) into (3.16) and solving for $\hat{\beta}_G$ (using equation 3.10):

$$\begin{aligned} \hat{\beta}_G &= -\theta C^{-1} B_{xz} B_{zz}^{-1} B_{zy} + C^{-1}(W_{xy} + \theta B_{xy}) \\ &= C^{-1} W_{xy} + \theta C^{-1}(B_{xy} - B_{xz} B_{zz}^{-1} B_{zy}) \\ &= C^{-1} W_{xx} \hat{\beta}_W + \theta C^{-1}(B_{xx} - B_{xz} B_{zz}^{-1} B_{zx}) \hat{\beta}_B \\ &= \Delta \hat{\beta}_W + (I_k - \Delta) \hat{\beta}_B \end{aligned}$$

since:

$$I_k - \Delta = I - C^{-1} W_{xx} = C^{-1}(C - W_{xx}) = \theta C^{-1}(B_{xx} - B_{xz} B_{zz}^{-1} B_{zx})$$

For α_G, substitute (3.17) into (3.16) again:

$$\begin{aligned}\hat{\alpha}_G &= B_{zz}^{-1}B_{zy} + \theta B_{zz}^{-1}B_{zx}C^{-1}B_{xz}B_{zz}^{-1}B_{zy} - B_{zz}^{-1}B_{zx}C^{-1}(W_{xy} + \theta B_{xy}) \\ &= B_{zz}^{-1}B_{zy} - B_{zz}^{-1}B_{zx}C^{-1}(W_{xy} + \theta(B_{xy} - B_{xz}B_{zz}^{-1}B_{zy})) \\ &= B_{zz}^{-1}B_{zy} - B_{zz}^{-1}B_{zx}\hat{\beta}_B\end{aligned}$$

which we compare to (3.11). □

The main point, conceptually, is that the error components (random effects) model implies coefficients that lie between the within and between coefficients. This somewhat surprising result is very useful, since one can construct the necessary matrices to get the GLS estimates by matrix-weighting the between and within estimates, and the weights are based on inverses of their corresponding variances. Practically, estimation can be done by estimating the variance components and then computing a corresponding weight, and running weighted least squares. There are a variety of ways to estimate the variance components, e.g. starting with the OLS residuals or the within residuals (since the OLS and within estimators are not biased, but only inefficient). Accordingly, Baltagi shows estimates for a variety of different random-effects estimation procedures, e.g. "WALHUS" for the method of Wallace and Hussain, "SWAR" for the method due to Swamy and Arora, "Amemiya" for Amemiya's method, and so on.

Looking ahead, the next major issue is specification tests, in particular for when to use fixed and when to use random effects. The workhorse test is due to Hausman. The main point of the Hausman test is that β_W is consistent under the null and alternative hypotheses, while β_G is consistent under the null, but has different probability limits when the null is rejected. So a statistic comparing the two is one way to test the suitability of fixed effects. Hausman and Taylor elaborated on the Hausman test, showing that the comparison between the within and between estimates is also useful (which is intuitive since the GLS estimator is a matrix weighted average of the two.) A large (and difficult) literature has built up further tests, and a very quick-and-dirty summary is that one should not stop with a Hausman test, even if it does not suggest rejecting the hypothesis of fixed effects.

Grunfeld Example (continued):

$I_{it} = \alpha + \beta_1 F_{it} + \beta_2 C_{it} + u_{it}$ w/ $u_{it} = \mu_i + \nu_{it}$

Baltagi's Table 2.1 (1st ed. only)

	β_1	β_2	$\theta=1-(\sigma_\nu/\sigma_1)^a$
OLS	0.116 (0.006)	0.231 (0.025)	0
Between	0.135 (0.029)	0.032 (0.191)	∞
Within	0.110 (0.012)	0.310 (0.017)	1
WALHUS	0.10973 (0.01029)	0.30765 (0.01724)	0.845779
AMEMIYA	0.10979 (0.01052)	0.30817 (0.01717)	0.863097
SWAR	0.10978 (0.01049)	0.30811 (0.01718)	0.861224
NERLOVE	0.10978 (0.01049)	0.30810 (0.01718)	0.860717
IMLE	0.10976 (0.01042)	0.30794 (0.01720)	0.855359

a. $\sigma_1 = \sqrt{T\sigma_\mu^2 + \sigma_\nu^2}$

(warning: theta above is not the same as in the $\TeX$ notes)
In this example, differences across the FGLS methods are minimal.

Baltagi's Table 2.1 (3rd ed.)

	β_1	β_2	ρ	σ_μ	σ_v
OLS	0.116	0.231			
	(0.006)	(0.025)			
Between	0.135	0.032			
	(0.029)	(0.191)			
Within	0.110	0.310			
	(0.012)	(0.017)			
WALHUS	0.110	0.308	0.73	87.36	53.75
	(0.011)	(0.017)			
AMEMIYA	0.110	0.308	0.71	83.52	52.77
	(0.010)	(0.017)			
SWAR	0.110	0.308	0.72	84.20	52.77
	(0.010)	(0.017)			
IMLE	0.110	0.308	0.70	80.30	52.49
	(0.010)	(0.017)			

$$\rho = \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\upsilon^2}$$ (so it is easy to compute from the other entries)

$$\Theta = 1 - \frac{\sigma_\upsilon}{\sqrt{T\sigma_\mu^2 + \sigma_\upsilon^2}}$$

STATA OUTPUT

[re option specifies random effects,
sa option specifies Swamy-Arora small-sample estimator—compare to SWAR]

```
.xtreg i f c, re i(firm) sa theta

Random-effects GLS regression                   Number of obs      =       200
Group variable (i): firm                        Number of groups   =        10

R-sq:  within  = 0.7668                         Obs per group: min =        20
       between = 0.8196                                        avg =      20.0
       overall = 0.8061                                        max =        20

Random effects u_i ~ Gaussian                   Wald chi2(2)       =    657.67
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000
theta              = .86122362

------------------------------------------------------------------------------
           i |      Coef.   Std. Err.       z     P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           f |   .1097811   .0104927    10.46    0.000      .0892159    .1303464
           c |    .308113   .0171805    17.93    0.000      .2744399    .3417861
       _cons |  -57.83441   28.89893    -2.00    0.045     -114.4753   -1.193537
-------------+----------------------------------------------------------------
     sigma_u |   84.20095
     sigma_e |  52.767964
         rho |  .71800838   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```


1

[attempt at iterative MLE—note didn't replicate 1st ed. se's exactly]

```
. xtreg i f c, re i(firm) mle

Fitting constant-only model:
Iteration 0:   log likelihood = -1387.6302
Iteration 1:   log likelihood = -1291.9897
Iteration 2:   log likelihood = -1254.2888
Iteration 3:   log likelihood = -1243.6309
Iteration 4:   log likelihood = -1242.0548
Iteration 5:   log likelihood = -1241.9709
Iteration 6:   log likelihood = -1241.9696
Iteration 7:   log likelihood = -1241.9696

Fitting full model:
Iteration 0:   log likelihood = -1105.6101
Iteration 1:   log likelihood = -1098.8418
Iteration 2:   log likelihood = -1095.4188
Iteration 3:   log likelihood = -1095.2576
Iteration 4:   log likelihood =  -1095.257

Random-effects ML regression                    Number of obs      =       200
Group variable (i): firm                        Number of groups   =        10
```

(over) ↓

13

Random effects u_i ~ Gaussian — Obs per group: min = 20; avg = 20.0; max = 20

LR chi2(2) = 293.43

Log likelihood = -1095.257 — Prob > chi2 = 0.0000

i	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
f	**.1097626**	**.010**3389	10.62	0.000	.0894988	.1300265
c	**.307942**	**.017**1006	18.01	0.000	.2744254	.3414585
_cons	-57.7672	27.70004	-2.09	0.037	-112.0583	-3.476114
/sigma_u	**80.297**29	18.37811	4.37	0.000	44.27685	116.3177
/sigma_e	**52.49**255	2.69306	19.49	0.000	47.21424	57.77085
rho	**.70**05943	.0985226			.4881266	.8603709

Likelihood-ratio test of sigma_u=0: chibar2(01)= 193.09 Prob>=chibar2 = 0.000

compute $\Theta = 1 - \frac{\sigma_v}{\sqrt{T\sigma_\mu^2 + \sigma_v^2}}$ = 1-(80.29729/(20(80.29729)^2+(52.492552)^2)

= 0.855359

14

[Wallace and Hussain, use OLS residuals to compute θ=0.8457797 see notes from last time for computation of ib, etc.]

```
. gen iwh=(i-0.8457797*ib)

. gen fwh=(f-0.8457797*fb)

. gen cwh=(c-0.8457797*cb)

. reg iwh fwh cwh

      Source |       SS       df       MS                  Number of obs =     200
-------------+------------------------------               F(  2,   197) =  329.96
       Model |  1858725.44     2  929362.719               Prob > F      =  0.0000
    Residual |   554863.18   197  2816.56437               R-squared     =  0.7701
-------------+------------------------------               Adj R-squared =  0.7678
       Total |  2413588.62   199   12128.586               Root MSE      =  53.071

------------------------------------------------------------------------------
         iwh |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
-------------+----------------------------------------------------------------
         fwh |   .1097338   .0102915     10.66    0.000       .0894382    .1300295
         cwh |   .3076469   .0172381     17.85    0.000        .273652    .3416417
       _cons |  -8.891503   4.080004     -2.18    0.030      -16.93759   -.8454122
------------------------------------------------------------------------------
```

est. $\sigma_v^2=\Sigma_i\Sigma_t(u_{it}-u_i.)^2/(N(T-1))$=586923.4/190=3089.1

15

[Amemiya, use within residuals to compute θ=0.863097]

```
. gen iam=(i-0.863097*ib)

. gen fam=(f-0.863097*fb)

. gen cam=(c-0.863097*cb)

. reg iam fam cam

      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F(  2,   197) =  328.71
       Model |  1829491.93     2  914745.966           Prob > F      =  0.0000
    Residual |  548223.542   197  2782.86062           R-squared     =  0.7694
-------------+------------------------------           Adj R-squared =  0.7671
       Total |  2377715.47   199   11948.319           Root MSE      =  52.753

------------------------------------------------------------------------------
         iam |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         fam |   .1097872   .0105167    10.44   0.000     .0890474    .1305269
         cam |   .3081661    .017174    17.94   0.000     .2742976    .3420345
       _cons |  -7.920601   4.002255    -1.98   0.049    -15.81336   -.0278391
------------------------------------------------------------------------------
```

16

Note that above, I "cheated" by taking the thetas from Baltagi's table and proceeding, not computing them directly. The ~~next~~ last set of notes do the job from scratch, correcting a mistake I originally made when trying to replicate theh "Amemiya" results.

(Part of) Baltagi's Table 3.1 shows estimates for two-way models:

$I_{it} = \alpha + \beta_1 F_{it} + \beta_2 C_{it} + u_{it}$ w/ $u_{it} = \mu_i + \lambda_t + \nu_{it}$

	β_1	β_2	θ_1	θ_2	θ_1
OLS	0.116	0.231	0	0	0
	(0.006)	(0.025)			
Within	0.11772	0.35792	1	1	1
	(0.01375)	(0.02272)			
WALHUS	0.10973	0.30757	0.8433	0	0
	(0.01026)	(0.01725)			
SWAR	0.10982	0.30845	0.87368	0	0
	(0.0165)	(0.01714)			
AMEMIYA	0.11159	0.32462	0.87475	0.29695	0.29595
	(0.01103)	(0.01885)			
IMLE	0.10990	0.30923	0.85595	0.02620	0.02612
	(0.01046)	(0.01735)			

[Two-Way Fixed Effects]

```
. anova i f c firm year, continuous (f c) reg

      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F( 30,   169) =  110.98
       Model |  8907796.87    30  296926.562           Prob > F      =  0.0000
    Residual |  452147.043   169  2675.42629           R-squared     =  0.9517
-------------+------------------------------           Adj R-squared =  0.9431
       Total |  9359943.92   199  47034.8941           Root MSE      =  51.725

------------------------------------------------------------------------------
           i        Coef.   Std. Err.       t     P>|t|     [95% Conf. Interval]
------------------------------------------------------------------------------
_cons           -53.58933   21.59303     -2.48   0.014     -96.21613   -10.96253
f                .1177158   .0137513      8.56   0.000      .0905694    .1448623
c                .3579163    .022719     15.75   0.000      .3130667    .4027659
firm
           1    -126.8371   58.52545     -2.17   0.032     -242.3722   -11.30197
           2     80.21714   29.92176      2.68   0.008      21.14857    139.2857
           3    -262.0679   29.61266     -8.85   0.000     -320.5263   -203.6095
           4    -31.48327   18.29614     -1.72   0.087     -67.60169    4.635146
           5    -132.2757   19.43931     -6.80   0.000     -170.6509   -93.90057
           6    -23.94848    17.0117     -1.41   0.161     -57.53129    9.634333
           7    -75.37051   17.70952     -4.26   0.000     -110.3309   -40.41014
           8     -59.3466   18.17438     -3.27   0.001     -95.22466   -23.46855
           9    -96.61956   17.63008     -5.48   0.000     -131.4231   -61.81601
          10    (dropped)
year
           1     93.52622   27.10786      3.45   0.001      40.01258    147.0399
           2     74.32881   26.28383      2.83   0.005       22.4419    126.2157
           3     52.83621   26.07413      2.03   0.044      1.363259    104.3092
           4     54.29982    26.0657      2.08   0.039      2.843504    105.7561
           5     24.05593   25.52497      0.94   0.347     -26.33292    74.44478
```

6	49.29115	25.4568	1.94	0.055	-.9631395	99.54543
7	74.72176	25.33737	2.95	0.004	24.70324	124.7403
8	72.38643	25.49756	2.84	0.005	22.05169	122.7212
9	50.5486	25.17805	2.01	0.046	.8446089	100.2526
10	50.42746	25.13577	2.01	0.046	.8069269	100.048
11	37.84319	24.95347	1.52	0.131	-11.41746	87.10384
12	62.35694	24.80931	2.51	0.013	13.38087	111.333
13	54.13398	24.7417	2.19	0.030	5.291383	102.9766
14	49.80971	24.63549	2.02	0.045	1.176774	98.44264
15	20.03112	24.43391	0.82	0.413	-28.20387	68.26612
16	17.63011	24.19119	0.73	0.467	-30.12572	65.38595
17	31.04532	23.67891	1.31	0.192	-15.69923	77.78987
18	28.89389	23.43739	1.23	0.219	-17.37387	75.16166
19	25.80826	23.22233	1.11	0.268	-20.03496	71.65148
20	(dropped)					

```
[create firm means as I did last time, and also year means]
. summarize i
. display r(mean)
. gen iw2=i-ibi-ibt+r(mean)

etc.

. reg iw2 fw2 cw2

      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F(  2,   197) =  253.47
       Model |  1163501.77     2  581750.886           Prob > F      =  0.0000
    Residual |  452147.055   197  2295.16272           R-squared     =  0.7201
-------------+------------------------------           Adj R-squared =  0.7173
       Total |  1615648.83   199  8118.83833           Root MSE      =  47.908

------------------------------------------------------------------------------
         iw2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         fw2 |   .1177158   .0127366     9.24   0.000     .0925983    .1428334
         cw2 |   .3579163   .0210426    17.01   0.000     .3164186     .399414
       _cons |  -2.36e-06   3.387597    -0.00   1.000    -6.680611    6.680607
------------------------------------------------------------------------------
```

again, wrong std. errors b/c actual d.f. is 200-3-(20-1)-(10-1)=169 not 197

$(0.0127366)(197/169)^{1/2}$ = **0.013751**
$(0.0210426)(197/169)^{1/2}$ = **0.022719**

Wallace and Hussain (first, take θs as given by Baltagi (note that 0s follow from negative estimates of variance components))

regression based on $y_{it} - \hat{\theta}_1 \bar{y}_{i\cdot} - \hat{\theta}_2 \bar{y}_{\cdot t} + \hat{\theta}_2 \bar{y}_{\cdot\cdot}$

```
. gen whi=i-0.84333*ibi

. gen whf=f-0.8433*fbi

. gen whc=c-0.8433*cbi

. reg whi whf whc

      Source |       SS       df       MS                  Number of obs =     200
-------------+------------------------------               F(  2,   197) =  330.15
       Model |  1863137.62     2  931568.811               Prob > F      =  0.0000
    Residual |  555870.045   197  2821.67535               R-squared     =  0.7702
-------------+------------------------------               Adj R-squared =  0.7679
       Total |  2419007.67   199  12155.8174               Root MSE      =  53.119

------------------------------------------------------------------------------
         whi |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
-------------+----------------------------------------------------------------
         whf |   .1097194   .0102588     10.70   0.000      .0894883     .1299505
         whc |   .3075698   .0172479     17.83   0.000      .2735556     .3415839
       _cons |  -9.033073    4.09138     -2.21   0.028     -17.1016     -.9645481
------------------------------------------------------------------------------
```

How to compute thetas? Goto **R** (with apologies for jumping around)

```
> grun <- read.csv("c:/documents/teaching/493Panel/grfldat.csv")
> y <- grun[,3]
> f <- grun[,4]
> c <- grun[,5]
> X <- grun[,4:5]
> X<- cbind(1,X)
>
> lm.OLS<-lm(y~f+c)
>
> e<- resid(lm.OLS)
>
> IN<-matrix(0,10,10)
> diag(IN)<-1
> IT<-matrix(0,20,20)
> diag(IT)<-1
> JbN<-matrix(0.1,10,10)
> JbT<-matrix(0.05,20,20)
> CN<-IN-JbN
> CT<-IT-JbT
>
> Q1<-kronecker(CN,CT)
> Q2<-kronecker(CN,JbT)
> Q3<-kronecker(JbN,CT)
> Q4<-kronecker(JbN,JbT)
>
> trQ1<-sum(diag(Q1))
> trQ2<-sum(diag(Q2))
> trQ3<-sum(diag(Q3))
> trQ4<-sum(diag(Q4))
>
> lam1<-(t(e)%*%Q1%*%e)/trQ1
> lam2<-(t(e)%*%Q2%*%e)/trQ2
> lam3<-(t(e)%*%Q3%*%e)/trQ3
> lam4<-(t(e)%*%Q3%*%e)/trQ4
>
> sigvsq<-lam1
```

```
> sigmusq<-(lam2-lam1)/200
> siglsq<-(lam3-lam1)/10
>
> theta1<-1-sqrt(sigvsq/lam2)
> theta2<-1-sqrt(sigvsq/lam3)
> theta3<-theta1+theta2+sqrt(sigvsq/lam4)-1
>
> lam1

          [,1]
 [1,] 3188.058
> lam2

          [,1]
 [1,] 129880.8
> lam3

          [,1]
 [1,] 2198.189
> lam4

          [,1]
 [1,] 41765.59
>
> sigvsq

          [,1]
 [1,] 3188.058
> sigmusq

          [,1]
 [1,] 633.4636
> siglsq

           [,1]
 [1,] -98.98689
```

```
>
> theta1

           [,1]
 [1,] 0.8433283
> theta2

            [,1]
 [1,] -0.2042886
> theta3

             [,1]
 [1,] -0.08467761
>
```

For Amemiya, repeat exactly, but use Within residuals not OLS residuals

Computing weights for Error Component Models (one more time)

Several of us tried, and failed, to get the θs reported by Baltagi for the Amemiya method of estimating one-way and two-way Error Component models. I went back to the 1971 *International Economic Review* article by Amemiya and confirmed that there's no difference (other than notation) between the matrices he uses to get the spectral decomposition of Ω and those Baltagi introduces. (Digression: I have that piece in an (expensive) Edward Elgar compilation of Amemiya articles. The autobiographical preface is extremely interesting, so I'll attach it here for your light-reading pleasure. Unlike Amemiya, I did not complete my doctoral thesis in one year.) Eventually (much too eventually!), I realized that the problem with my calculations was with the "within residuals." Amemiya proposes using the within estimates to compute residuals as such (for the one-way version):

$$\tilde{u} = y - \tilde{\alpha}\mathbf{1}_{NT} - X\tilde{\beta} \quad \text{where} \quad \tilde{\alpha} = \bar{y}_{..} - \bar{X}'_{..}\tilde{\beta}$$

The problem with using the actual residuals from the Within model in computing variance components is that it assures an estimate that is effectively 0 for $\hat{\sigma}_1^2$, since the Within transformation creates residuals that sum to zero within each group (firm in this case).

Confusing matters still more, I also eventually noticed that the two editions of Baltagi's text have slightly different values for θ in Table 2.1, the results on Grunfeld's data for the one-way model.

Baltagi's Table 2.1 (1st ed.) $I_{it} = \alpha + \beta_1 F_{it} + \beta_2 C_{it} + u_{it}$ w/ $u_{it} = \mu_i + \nu_{it}$

	β_1	β_2	$\theta = 1-(\sigma_\nu/\sigma_1)^a$
OLS	0.11556	0.23068	0
	(0.00584)	(0.02548)	
Between	0.13465	0.03203	∞
	(0.02875)	(0.19094)	
Within	0.11012	0.31007	1
	(0.01186)	(0.01735)	
WALHUS	0.10973	0.30765	0.845779
	(0.01029)	(0.01724)	
AMEMIYA	0.10979	0.30817	0.863097
	(0.01052)	(0.01717)	
SWAR	0.10978	0.30811	0.861224
	(0.01049)	(0.01718)	
. . .			

a. $\sigma_1 = \sqrt{T\sigma_\mu^2 + \sigma_\nu^2}$

Baltagi's Table 2.1 (2nd ed.) $I_{it} = \alpha + \beta_1 F_{it} + \beta_2 C_{it} + u_{it}$ w/ $u_{it} = \mu_i + v_{it}$

	β_1	β_2	$\theta=1-(\sigma_v/\sigma_1)^a$
OLS	0.116	0.231	0
	(0.006)	(0.025)	
Between	0.135	0.032	∞
	(0.029)	(0.191)	
Within	0.110	0.310	1
	(0.012)	(0.017)	
WALHUS	0.110	**0.307**	**0.837**
	(0.011)	**(0.018)**	
AMEMIYA	0.110	0.308	**0.856**
	(0.010)	(0.017)	
SWAR	0.110	0.308	0.861
	(0.010)	(0.017)	
. . .			

a. $\sigma_1 = \sqrt{T\sigma_\mu^2 + \sigma_v^2}$

bold marks values that changed from 1st ed. (other changes may be hidden by rounding)

2005 update: the 3rd edition dropped the θ column in favour of ρ, σ_μ and σ_v and rounded the values at 2 or 3 decimal places, like the 2nd ed. and unlike the 1st. Surprisingly, there are still minor discrepancies!

	WALHUS			AMEMIYA		
	1st ed.	2nd ed.	3rd ed.	1st ed.	2nd ed.	3rd ed.
β_1	0.10973	0.110	0.110	0.10979	0.110	0.110
se(β_1)	0.01029	0.011	0.011	0.01052	0.010	0.010
β_2	0.30765	0.307	0.308	0.30817	0.308	0.308
se(β_2)	0.01724	0.018	0.017	0.01717	0.017	0.017
θ	0.845779	0.837	0.8637*	0.863097	0.856	0.8601*
ρ			0.73			0.71
σ_μ			87.36			83.52
σ_v			53.75			52.77

* computed from (rounded) rho and sigma values

Until I noticed the discrepancy between editions, I was obsessing with trying to replicate the 1st edition numbers. Fortunately, with the correct method of computing "within residuals," I can replicate the revised (2nd ed.) results.

2005: you don't have these

Below, I also correct the two errors that were in the **R** code in my April 14 notes, a "200" where I wanted "20" (because it was T), and the failure to replace negative values of certain parameter estimates with 0. (Neither of those mistakes had consequences for estimating θ_1, the only non-zero weight under the Wallace-Hussain method, so they were easy to miss.) I also compute the OLS and within residuals directly in **R** rather than importing residuals saved in STATA.

First, the one-way model. Here:

$$\mathbf{\Omega}^{-1/2} = \frac{1}{\sigma_1}\mathbf{P} + \frac{1}{\sigma_\upsilon}\mathbf{Q} \quad \text{where } \mathbf{P} = \mathbf{I}_N \otimes \bar{\mathbf{J}}_T \text{ and } \mathbf{Q} = \mathbf{I}_{NT} - \mathbf{P}$$

$$\hat{\sigma}_1^2 = \frac{\mathbf{u'Pu}}{tr(\mathbf{P})} \quad \text{and} \quad \hat{\sigma}_\upsilon^2 = \frac{\mathbf{u'Qu}}{tr(\mathbf{Q})}$$

For the **u** vector we substitute an estimate, either $\hat{u}_{OLS}$ or $\tilde{u}$, the within residuals defined above.

```
> grun <- read.csv("c:/documents/teaching/493Panel/grfldat.csv")
>
> y <- grun[,3]
> firm <- grun[,1]
> f <- grun[,4]
> c <- grun[,5]
> X <- grun[,4:5]
> X <- cbind(1,X)
>
> IN<-matrix(0,10,10)
> diag(IN)<-1
> IT<-matrix(0,20,20)
> diag(IT)<-1
> INT<-kronecker(IN,IT)
> JbN<-matrix(0.1,10,10)
> JbT<-matrix(0.05,20,20)
> CN<-IN-JbN
> CT<-IT-JbT
>
> P<-kronecker(IN,JbT)
> Q<-INT-P
> trP<-sum(diag(P))
> trQ<-sum(diag(Q))
>
>
> lm.ols <-lm(y~f+c)
> e<-resid(lm.ols)
>
> firm<-factor(firm)
> lm.win <-lm(y~f+c+firm)
> u<-resid(lm.win)
>
> Q <- as.matrix(Q)
> X <- as.matrix(X)
> betaw<-solve((t(X)%*%Q%*%X))%*%t(X)%*%Q%*%y
>
```

```
> alphaw<-mean(y)-mean(f)*betaw[2,]-mean(c)*betaw[3,]
> u2<-y-alphaw-f*betaw[2,]-c*betaw[3,]
>
> # Wallace and Hussain method with OLS residuals
> # matches 2nd ed Baltagi results
> #
> sig1sq1<-(t(e)%*%P%*%e)/trP
> signusq1<-(t(e)%*%Q%*%e)/trQ
> sigmusq1<-(sig1sq1-signusq1)/20
>
> sig1sq1

          [,1]
 [1,] 116892.7
> signusq1

          [,1]
 [1,] 3089.071
> sigmusq1

          [,1]
 [1,] 5690.182
>
> theta1<-1-(sqrt(signusq1)/sqrt(sig1sq1))
>
> theta1

           [,1]
 [1,] 0.8374376
>
>
> # Amemiya method from residuals from within
> # produces estimate of 0 for theta
> #
> sig1sq2<-(t(u)%*%P%*%u)/trP
```

```
> signusq2<-(t(u)%*%Q%*%u)/trQ
> sigmusq2<-(sig1sq2-signusq2)/20
>
> sig1sq2

                [,1]
  [1,] 1.316047e-28
> signusq2

            [,1]
  [1,] 2755.148
>
> theta2<-1-(sqrt(signusq2)/sqrt(sig1sq2))
>
> theta2

                 [,1]
  [1,] -4.575481e+15
>
> # Amemiya method from computed within residuals
> # matches 2nd ed Baltagi results
> #
> sig1sq2<-(t(u2)%*%P%*%u2)/trP
> signusq2<-(t(u2)%*%Q%*%u2)/trQ
> sigmusq2<-(sig1sq2-signusq2)/20
>
> sig1sq2
          [,1]
[1,] 132301.1
> signusq2
          [,1]
[1,] 2755.148
>
> theta2<-1-(sqrt(signusq2)/sqrt(sig1sq2))
>
> theta2
```

```
      [,1]
[1,] 0.8556919
>
```

Success for the one-way model! Now for the two-way model, which is the one that was tripping us up to begin with…

Here, $u_{it} = \mu_i + \lambda_t + \upsilon_{it}$ and:

$$\mathbf{\Omega} = \sum_{i=1}^{4} \gamma_i \mathbf{Q}_i \text{ and } \sigma_v \mathbf{\Omega}^{-1/2} = \sum_{i=1}^{4} \left(\sigma_v \middle/ \sqrt{\gamma_i} \right) \mathbf{Q}_i$$

(I'm substituting gamma for the lambdas I had originally, since lambda is already representing the time effects in the residual term.)

$$\gamma_1 = \sigma_v^2, \quad \gamma_2 = T\sigma_\mu^2 + \sigma_v^2, \quad \gamma_3 = N\sigma_\lambda^2 + \sigma_v^2, \quad \gamma_4 = N\sigma_\lambda^2 + T\sigma_\mu^2 + \sigma_v^2$$

and $\hat{\gamma}_i^2 = \dfrac{\mathbf{u}_i' \mathbf{Q}_i \mathbf{u}_i}{tr(\mathbf{Q}_i)}$

Finally, $\theta_1 = 1 - \left(\sigma_v \middle/ \sqrt{\gamma_2} \right)$, $\theta_2 = 1 - \left(\sigma_v \middle/ \sqrt{\gamma_3} \right)$, $\theta_3 = \theta_1 + \theta_2 + \left(\sigma_v \middle/ \sqrt{\gamma_4} \right) - 1$

(Part of) Baltagi's Table 3.1 (1st ed.) shows estimates for the two-way models:

$I_{it} = \alpha + \beta_1 F_{it} + \beta_2 C_{it} + u_{it}$ w/ $u_{it} = \mu_i + \lambda_t + v_{it}$

	β_1	β_2	θ_1	θ_2	θ_1
OLS	0.11556	0.23068	0	0	0
	(0.00584)	(0.02548)			
Within	0.11772	0.35792	1	1	1
	(0.01375)	(0.02272)			
WALHUS	0.10973	0.30757	0.8433	0	0
	(0.01026)	(0.01725)			
AMEMIYA	0.11159	0.32462	0.87475	0.29695	0.29595
	(0.01103)	(0.01885)			
SWAR	0.10982	0.30845	0.87368	0	0
	(0.0165)	(0.01714)			

This time, the 2nd ed. values are, again, rounded, but are consistent for the WALHUS and AMEMIYA rows. The SWAR theta-1 in the 2nd ed., however, has become 0.864. (2005: the 3rd edition doesn't report the thetas.)

```
> grun <- read.csv("c:/documents/teaching/493Panel/grfldat.csv")
>
> firm <- grun[,1]
> year <- grun[,2]
> y <- grun[,3]
> f <- grun[,4]
> c <- grun[,5]
> X <- grun[,4:5]
> X<- cbind(1,X)
>
> IN<-matrix(0,10,10)
> diag(IN)<-1
> IT<-matrix(0,20,20)
> diag(IT)<-1
> JbN<-matrix(0.1,10,10)
> JbT<-matrix(0.05,20,20)
> CN<-IN-JbN
> CT<-IT-JbT
>
> Q1<-kronecker(CN,CT)
> Q2<-kronecker(CN,JbT)
> Q3<-kronecker(JbN,CT)
```

```
> Q4<-kronecker(JbN,JbT)
>
> trQ1<-sum(diag(Q1))
> trQ2<-sum(diag(Q2))
> trQ3<-sum(diag(Q3))
> trQ4<-sum(diag(Q4))
>
> lm.ols <-lm(y~f+c)
> e<-resid(lm.ols)
>
> firm<-factor(firm)
> year<-factor(year)
> lm.win <-lm(y~f+c+firm+year)
> u<-resid(lm.win)
>
> Q1 <- as.matrix(Q1)
> X <- as.matrix(X)
> betaw<-solve((t(X)%*%Q1%*%X))%*%t(X)%*%Q1%*%y
>
> alphaw<-mean(y)-mean(f)*betaw[2,]-mean(c)*betaw[3,]
> u2<-y-alphaw-f*betaw[2,]-c*betaw[3,]
>
>
```

```
> gam1<-(t(e)%*%Q1%*%e)/trQ1
> gam2<-(t(e)%*%Q2%*%e)/trQ2
> gam3<-(t(e)%*%Q3%*%e)/trQ3
>
> gam1w<-(t(u)%*%Q1%*%u)/trQ1
> gam2w<-(t(u)%*%Q2%*%u)/trQ2
> gam3w<-(t(u)%*%Q3%*%u)/trQ3
>
> gam1w2<-(t(u2)%*%Q1%*%u2)/trQ1
> gam2w2<-(t(u2)%*%Q2%*%u2)/trQ2
> gam3w2<-(t(u2)%*%Q3%*%u2)/trQ3
>
> sigvsq<-max(gam1,0)
> sigmusq<-max((gam2-gam1)/20,0)
> siglsq<-max((gam3-gam1)/10,0)
>
> sigvsqw<-max(gam1w,0)
> sigmusqw<-max((gam2w-gam1w)/20,0)
> siglsqw<-max((gam3w-gam1w)/10,0)
>
> sigv2w2<-max(gam1w2,0)
> sigmu2w2<-max((gam2w2-gam1w2)/20,0)
> sigl2w2<-max((gam3w2-gam1w2)/10,0)
```

```
>
> gam4<-20*sigmusq+10*siglsq+sigvsq
> gam4w<-20*sigmusqw+10*siglsqw+sigvsqw
> gam4w2<-20*sigmu2w2+10*sigl2w2+sigv2w2
>
> theta1<-max(1-sqrt(sigvsq/gam2),0)
> theta2<-max(1-sqrt(sigvsq/gam3),0)
> theta3<-max(theta1+theta2+sqrt(sigvsq/gam4)-1,0)
>
> theta1w<-max(1-sqrt(sigvsqw/gam2w),0)
> theta2w<-max(1-sqrt(sigvsqw/gam3w),0)
> theta3w<-max(theta1w+theta2w+sqrt(sigvsqw/gam4w)-1,0)
>
> theta1w2<-max(1-sqrt(sigv2w2/gam2w2),0)
> theta2w2<-max(1-sqrt(sigv2w2/gam3w2),0)
> theta3w2<-max(theta1w2+theta2w2+sqrt(sigv2w2/gam4w2)-1,0)
>
> gam1

         [,1]
 [1,] 3188.058
> gam2
```

```
         [,1]
 [1,] 129880.8
> gam3

         [,1]
 [1,] 2198.189
> gam4
[1] 129880.8
>
> sigvsq
[1] 3188.058
> sigmusq
[1] 6334.636
> siglsq
[1] 0
>
> theta1
[1] 0.8433283          ← matches Baltagi's table, OLS row
> theta2
[1] 0                  ← matches Baltagi's table, OLS row
> theta3
[1] 0                  ← matches Baltagi's table, OLS row
>
```

```
>
> gam1w

         [,1]
 [1,] 2644.135
> gam2w

             [,1]
 [1,] 2.165697e-28
> gam3w

             [,1]
 [1,] 1.764486e-28
> gam4w
[1] 2644.135
>
> sigvsqw
[1] 2644.135
> sigmusqw
[1] 0
> siglsqw
[1] 0
>
```

```
> theta1w
[1] 0            ← bad estimate from using within residuals
> theta2w
[1] 0            ← bad estimate from using within residuals
> theta3w
[1] 0            ← bad estimate from using within residuals
>
> gam1w2
      [,1]
[1,] 2644.135
> gam2w2
      [,1]
[1,] 168538.5
> gam3w2
      [,1]
[1,] 5349.423
> gam4w2
[1] 171243.7
>
> sigv2w2
[1] 2644.135
> sigmu2w2
[1] 8294.716
```

> sigl2w2
[1] 270.5288
>
> theta1w2
[1] **0.8747458** ← **matches Baltagi's table, Amemiya θ_1=0.875**
> theta2w2
[1] **0.2969466** ← **matches Baltagi's table, Amemiya θ_2=0.297**
> theta3w2
[1] **0.2959532** ← **matches Baltagi's table, Amemiya θ_3=0.296**
>

Studies in Econometric Theory

The Collected Essays of Takeshi Amemiya

Takeshi Amemiya
Edward Ames Edmonds Professor of Economics
Stanford University, US

Edward Elgar

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Introduction

On April 26, 1941 I sailed from Yokohama on the N.Y.K. liner *Asama-maru* for Lima, Peru, with my mother and sister, to join my father, who was already in Lima working for the N.Y.K. office there. I had just turned six. (Many of the details of the subsequent account of the trip to and from Peru are based on the diary of my late sister Hiroko who was twenty years old at the time.) We had a suite on the C deck, and everything was luxurious on the ship: food, games, a swimming pool, and parties. The first Wrigley's chewing-gum I tasted on the ship had a strong impression on me. After stopping over in Honolulu one day, the ship arrived in San Francisco in the early morning of May 10. I still vividly remember looking up at the Golden Gate Bridge from a cabin window as the ship sailed under it. The bridge was only four years old then. No one could have guessed at that time that I would sail again under the bridge seventeen years later as a student and drive over it numerous times as an adult. We stayed in Yamato Hotel on California St. for five nights. The hotel is no longer there. I remember seeing in a street near the hotel a spot which still showed remnants of the damage caused by the 1906 earthquake. One day we crossed the Bay Bridge (then five years old) to visit the University of California at Berkeley. Unfortunately, we did not visit Stanford University. On May 15 we boarded the N.Y.K. freighter *Ginyo-maru*. After stopping over at Los Angeles, Manzanillo, Acapulco, Balboa and Buenaventura for freight loading and unloading, the *Ginyo-maru* arrived in Callao, the port of Lima, on June 13.

Our life in Lima was comfortable and pleasant until that fateful day, December 7. We lived in a spacious two-storey house in a central residential area of Lima called Miraflores. I attended the first grade of a Japanese school. In spite of its name, all the subjects except the Japanese language were taught in Spanish. As a result, my grades were all B's (generous ones) except A's in music and mathematics, which did not require so much knowledge of Spanish. (Nor of Japanese, I should say, for I got a B in Japanese as well.) I remember with nostalgia the trips to and from school every day with several neighbourhood kids squeezed in a small old car driven by a school chauffeur, the unforgettable smell of the corner bakery and candy store, the four-day carnival in February during which one could hit anybody of the opposite sex with balloons filled with water, and peddlers who came to sell all sorts of food with melodious calls.

On December 7, 1941 the Pacific war broke out. Late in that evening I was awakend by an unusual noise coming from downstairs, and I found my family huddled around a huge old-fashioned radio we had in the saloon, listening intensely to a short-wave broadcast from Japan announcing the outbreak of war in the Pacific. I can still recall the atmosphere of tension and anxiety that filled the room.

After that day our life remained at least superficially normal for a while, even though my parents and my sister, who was old enough to understand, must have been quite concerned about our future. The matter became more urgent on January 24, 1942, when Peru suddenly severed its diplomatic relations with Japan. It was the first country in South America to do so. Immediately the Peruvian government started rounding up influential Japanese, diplomats and senior employees of major Japanese firms as well as immigrants who held important positions, for interrogation. My father was taken away to a barrack in Callao the next day and was detained there until March 28. Although this was obviously a period of great anxiety, we did not suffer any harassment or feel any animosity from the Peruvians we met in our ordinary lives. In the meantime an agreement was reached between Japan and Peru that all the Japanese nationals were to be sent home.

On April 14 we embarked on a long travel home aboard a passenger ship called the *Acadia*. It was a relatively small ship (9,000 tons) and on its side the word DIPLOMATIC was painted in huge white letters. The ship was extremely crowded with diplomats and businessmen of Japanese, German and Italian nationality. The diplomats occupied the better quarters; my father slept on a hammock in a crowded cabin with many other men, and my mother, my sister and I were quartered in a windowless muggy cabin of approximately 12 by 12 feet. The food was reasonable, and the crew was efficient and kind. The *Acadia* stopped at Guayaquil, Ecuador and Buenaventura, Columbia to pick up more diplomats, and passing through Panama Canal and crossing the Gulf of Mexico, arrived in the port of New Orleans on April 24.

The next morning only the diplomats were allowed to leave the ship, and the rest of us had to wait aboard until after dinner without knowing where we would be taken from there. Finally, at about nine in the evening we started disembarking, and by looking at the luggage tags someone had put on our trunks we found that Seagoville, Texas was to be our next destination. Seagoville is a small town, about ten miles southeast of Dallas; we were to stay in a relocation camp transformed from a prison for women. Our train left the New Orleans station at about 10 p.m. and arrived in Dallas on the morning of the second day. The train was comfortable and the meals in the dining car were much better than on the *Acadia*. The waiter gave me a second helping of shrimp potage for lunch. At the Dallas station we got on two Greyhound buses and arrived at the camp in about forty minutes.

The camp impressed me as a bright, comfortable place, with many two-storey buildings scattered on spacious grounds. Around each building there were well-maintained lawns, flower beds and walking paths paved with bricks. The building we lived in looked like a college dorm. My father and my sister each had their own room, and my mother and I occupied another room. Each room was small, with the minimum necessities, but comfortable. I have mostly good memories of life in Seagoville: oatmeal at breakfast, many friends to play with all day long, big delicious oranges as many as we could eat, colourful lollipops we could buy at the canteen, and the beautiful sunset in the big Texas sky. The only bad experience I had was when I was confined in a hospital room with a cold and had to keep shouting "yo quiero agua" until finally a nurse who understood Spanish came by. Most of the Americans who worked on the premises, all of whom were women because of the former nature of the place, were nice to us. From what I later found out, our experience in Seagoville was far better than that of the Japanese Americans in their internment camps.

A major source of anxiety for the adults during our stay in Seagoville was, however, that for a long while we were uninformed about when and how we would get to Japan. Only in early June did we start hearing about the plan for "exchange ships", which would carry the Japanese from the United States and the Americans from Japan and exchange them at Lourenço Marques (now called Maputo) in

Mozambique (then a colony of Portugal, which was a neutral country during W.W.II).

The day of departure finally came on June 9, after forty-three days of waiting. We went back to Dallas on a bus and in the evening got on a train for New York. After spending two nights on the train, we arrived at Pennsylvania Station in New York City at around noon. The only thing I remember of this train ride is the long tunnel under the Hudson River right before the train arrived in New York City. From the station we were escorted by many policemen to the Pennsylvania Hotel (now called New York Penta) across 7th Avenue. After having lunch in a restaurant on the top floor of the 18-storey hotel, my family was taken to a luxurious suite on the fourth floor. Even to this day that is the most luxurious hotel room I have ever stayed in. The only fly in the ointment was that every match box in the room bore the inscription "Remember Pearl Harbor". During our one week stay in the hotel, we were not allowed to go outside, except once when we were taken to the rooftop, from which I remember seeing the Empire State Building very close by. Although we could not go shopping ourselves, we were able to ask hotel employees to buy a few things for us. In that way we bought a few boxes of Hershey's Kisses, which we took back to Japan. They became a family treasure back home during the war.

Suddenly on the morning of June 18, the officers announced that we were to board the ship that day. A bus took us to Jersey City across the Hudson River, where we boarded the *Gripsholm*, a Swedish ship weighing 17,000 tons. At 11.25 p.m. on June 18, 1942, the *Gripsholm* departed from Jersey City carrying 1,500 Japanese diplomats and businessmen and their families. On the ship were many famous people, some of whom I got to know well after I became an adult. They included an internationally acclaimed xylophonist Yoichi Hiraoka, for whom my sister played piano accompaniments in many concerts on the ship; Shigeto Tsuru, a classmate of Paul Samuelson's at Harvard who later became my colleague at Hitotsubashi University in Tokyo; Hachiro Yuasa, Ms. Kiyoko Cho and Ms. Tane Takahashi, who later became the first president, a professor of history of thought, and the head librarian, respectively, at the International Christian University, where I obtained my B.A. degree; Shizuo Kakutani, a leading Japanese mathematician famous among economists by his fixed-point theorem.

The *Gripsholm* arrived in Rio de Janeiro on July 3, took in more repatriates, and departed the next day. On July 20 she reached her final destination, Lourenço Marques. The next day we went shopping in the city, stepping on the earth for the first time in a little over a month. The only thing I remember about Lourenço Marques is that my parents bought me the best lollipops I have ever tasted. On July 22 the *Asama-maru* and the Italian ship *Conte Verde* arrived from Japan carrying the repatriates going back to the United States and other Allied countries. It was most exciting for the Japanese aboard the *Gripsholm*, especially for us because we sailed on the *Asama-maru* a year ago, to discover the *Asama-maru* from afar and watch her familiar figure loom larger and larger as she approached us. So many passengers gathered on one side of our ship to watch and greet the *Asama-maru* that stewards came to warn us that the ship had started to lean toward one side and urged some of the people to move to the other side.

July 23 was the day of the exchange. A long row of freight cars was placed on the railroad tracks alongside the pier, and the Americans were to walk on one side of the cars, and the Japanese on the other side. I have later heard that despite such a precaution some people crossed the tracks between cars and greeted their old friends, who were now on the opposing side of the conflict. The Japanese from North

America boarded the *Asama-maru*, and those from Middle and South America boarded the *Conte Verde*. The day after we boarded the *Conte Verde* we went shopping in the city again; in a store we came across a few friendly Americans, who had come out to the city from the *Gripsholm*, and we exchanged our U.S. dollars with their Japanese yen.

The two East-bound ships departed from Lourenço Marques on July 26 and sailed alongside of each other toward home. I enjoyed my life on the *Conte Verde*. Our cabin was comfortable and the meals were quite good. Except for a two-hour study period, we had a lot of fun running around on the decks and up and down the stairs over the several levels of the big ship as if going through a maze. In addition, there were movies, concerts and even an athletic meet on the deck. After stopping over two nights in Singapore, we came back to Yokohama on August 20. I cried, not wanting to leave the ship.

I must hurry through the rest of my life story, as it is of less historical significance. The foregoing journey, although consciously remembered mostly by Wrigley's chewing-gums, Hershey's Kisses and lollipops, must have registered a much deeper impact on the subconscious. A desire to go to the United States gradually strengthened in my mind during my adolescence. I chose the International Christian University (ICU) partly for the purpose of satisfying this desire.

A plan to found an international university based on Christian beliefs had been formulated many years before the war, but it gained momentum after the war through a joint effort of Japanese and American Protestant Christians. One of the leaders of this project was Hachiro Yuasa, my fellow *Gripsholm* voyager, who became the first president of ICU. ICU, patterned after a small liberal arts college in the United States, admitted its first class in 1953 and I entered it in the following year.

There were several international (mostly American) faculty members, and lectures were given both in Japanese and English. Students were expected to become bilingual; in the first year the Japanese students studied almost solely English, and the foreign students, Japanese. By the time I graduated from high school, I was able to read most difficult English books, but my hearing and speaking were extremely deficient. By the end of my first year at ICU, I could truly say I was fluent in English. I majored in economics, not because I wanted to study the subject seriously but because in Japan that was the subject customarily chosen by a student who did not have a clear idea of what field he or she wanted to specialize in. Indeed, I did not study economics seriously and spent more time reading American literature.

Even though my desire to go to America was intense, I did not start contemplating any concrete plan to accomplish this goal until the deadline for applying to graduate schools approached. When I sought advice, Prof. Alan Gleason, who to this day has remained my revered teacher and friend, looking at my less-than-B-average grades, rightly advised against my going to graduate school right after graduating from ICU. Following his advice, I applied and was admitted to Guilford College, a small Quaker college in North Carolina, as a nonmatriculate special student. I was recommended to Guilford by Prof. Iwao Ayusawa, a Quaker and an internationally known scholar of labour relations.

On August 3, 1958 I sailed from Yokohama aboard the N.Y.K. freighter *Shiga-maru*. I was somewhat apprehensive about my uncertain future because I still did not have a concrete idea about what to do after one year in Guilford. I entertained a vague possibility of coming back after one year and working for an English-language newspaper in Japan. At any rate I did not want to think too far ahead. I would take one year at a time.

The life on a freighter was very pleasant. The ship carried only twelve passengers, and there was a friendly, family-like atmosphere. It was a long voyage, stopping at San Francisco and Los Angeles, going through the Panama Canal, all the way to New York City, arriving there on August 30. It was for me a nostalgic return to San Francisco and New York City. I enjoyed sight-seeing in New York for eleven days and in Washington, D.C. for two days, each time staying in the home of my parents' friends. On September 11 I got on a Southern Railway night train heading for Greensboro, North Carolina, and became all alone for the first time. The train arrived in Greensboro early the next morning, where I was met by someone who took me to Guilford College only a few minutes away. The first two days I stayed with Prof. and Mrs. Algie Newlin, friends of the Ayusawas, and then moved to a dormitory.

My year at Guilford was an enjoyable one. I owe much of this to the warm hospitality of the Newlins on my frequent visits to their home and to my friendship with my roommate Jeff Hartsell. I was also fortunate to meet my English teacher Dr. Chauncey Ives and his wife. My and later my family's friendship with the Ives continued until Dr. Ives' death in 1992. (Mrs. Ives had died a few years earlier.) Dr. Ives was a man of exceptional intelligence and courage. He had a law degree from Yale and a Ph.D. in English literature from Harvard. His first teaching job was at the University of North Carolina, from which he was fired because he failed several football players. Then he moved to Guilford College, from which he was fired again soon after I left Guilford because of his passionate crusade to end segregation there. Afterwards he taught at a small women's college in New Jersey until his retirement.

The year I was in Guilford, the equal-rights movement for the blacks under the leadership of Dr. Martin Luther King, Jr. was just gaining momentum. I was in the midst of a radical change in the American South. In 1958 most of the schools in the South including Guilford were segregated; so were buses, restaurants and public rest rooms. All this quickly changed soon after I left Guilford, beginning with the famous sit-in strike at a Greensboro diner.

It is important to recognize that there is much more to the difference between the North and the South than in their attributed attitudes toward race. Sometimes I detect in the mind of the élite of the Eastern establishment irrational contempt and aversion toward the South. The Japanese students who study in an Eastern university and go home without experiencing the other cultures within the United States tend to be influenced by this attitude. I am glad I went to the South first, to become free of this bias. When I went to Guilford, it was only 13 years after the end of the W.W.II, and I was a little bit anxious about how Americans felt about the Japanese. There was no reason to worry because the Southerners remembered the War between the States more vividly than W.W.II! One student said, half jokingly and half seriously, "I hope you will be on our side when we rise again."

I was fortunate to have Dr. E. Kidd Lockard as my teacher of economics at Guilford. I took his courses on Money and Banking and Public Finance, and these subjects aroused so much interest in me that toward the end of the year at Guilford I decided to apply for graduate schools. My plan at that time was to obtain an M.A. in economics and then go home and work for a newspaper company. I was accepted by the American University in Washington, D.C. with a small scholarship ($350 a year as I remember), so I went there in the fall of 1959. All the graduate courses in the American University were offered in the evening starting at 7 p.m. to benefit those who worked in the daytime in U.S. government or international agencies. I was able to sustain myself for two years by working in the school library in the daytime, with the help of the scholarship and the savings from a job as a camp counsellor during the two consecutive summers.

While attending the American university, I lived in the home of Mr. and Mrs. Raymond Wilson, friends of both the Ayusawas and the Newlins. Living with the Wilsons was a far better education than attending any university. Mr. Wilson was a founder and Executive Secretary of the Friends Committee on National Legislation, a Quaker lobbyist group, and was a man truly dedicated to world peace. The fact that Mr. Wilson never received a Nobel Peace Prize has made me regard that prize in very low esteem. Mr. Wilson took me to Congress and to conferences on peace. In one of these conferences I heard a speech by Dr. Kenneth Boulding, a noted economist and a pacifist, and was greatly impressed by it. I also learned a great deal from his graduate textbook *Economic Analysis*. In 1991 I had the honor of meeting him in person at the University of Colorado. Mrs. Wilson was an extremely charming person whom everybody loved. She was like a second mother to me.

The two years in Washington were fruitful academically, as well. I was exposed to rigorous economic theory for the first time in the course on Soviet economy taught by a young professor named Dr. Bowles. His stimulating lectures had a great deal to do with my eventual decision to go on toward a Ph.D. At the same time I gave up the idea of going into journalism and started contemplating a teaching position after obtaining a Ph.D. In my second year at the American University, I applied to several graduate schools. My main interest in economics in those days was in the areas of Soviet economy and socialist economics; therefore, my first choice was Stanford University, where Paul Baran taught. But, ironically, I was rejected by Stanford and so went to Johns Hopkins University, which offered me a fellowship. One never knows what is one's fortune and misfortune. Paul Baran lived only for one more year and I went to Stanford as an assistant professor in three years; most of the students who were admitted to Stanford in the same year I was rejected were still there.

After obtaining a Master's degree from the American University, I went back to Japan for the first time in three years. I took a train from Washington, D.C. to Los Angeles and sailed from there on a freighter. It was a most exciting moment when Japanese land gradually came into view. You do not get the same feeling if you go by air. After a three-months vacation in Japan, I sailed again from Yokohama to San Francisco and then took a train to Baltimore. This was the third and last time I crossed the Pacific by ship, for soon afterward the N.Y.K. discontinued the service of carrying passengers on its freighters.

I am glad I went to Johns Hopkins. One gets a truly first-class education there because of the small size of the department and the very close tie among the faculty and students. One of its attractive features was that all the faculty members and all the graduate students attended a weekly seminar. At first, seminars were frightening because the participants seemed to take special pleasure in using jargon I had never heard before, such as multicollinearity and heteroscedasticity. As time went by, however, they turned out to be an excellent educational process. Taking Richard Musgrave's course on public finance was another excellent educational process. I learned more economics from his well-known textbook *Public Finance* than from any other book. Another attractive feature of Johns Hopkins was that professors were easily accessible to graduate students. Most notably, Prof. Edwin Mills always kept his office door open and welcomed me whenever I wanted to speak to him.

After completing the preliminary exams in the first year, I became a research assistant to Prof. Mills in his project on the water resources development in the State of Maryland because of my interest in economic planning and policy. But later I found out that in order to do serious research in this area you have to know the chemistry of water well, and after attending the geology library continuously for about one month I became quite bored by hydrology and my morale was weakened. So at the end of the second year Prof. Mills kindly relieved me of my duty in his project, and at about the same time, following a suggestion of Prof. Carl Christ, applied for a Ford Foundation dissertation fellowship in the area of econometric theory. At around this time I was beginning to be somewhat disillusioned by economic theory in general, and I felt that by studying statistics I could do useful things for a wider range of problems.

Having received the Ford Foundation fellowship, I started my dissertation research in econometric theory in my third year at Hopkins, with Prof. Christ as the main advisor. The other members of the thesis committee were Prof. Mills and Prof. Geoffrey Watson of the Statistics Department. At first it seemed unlikely that I would be able to complete the thesis in one year. When I learned in the middle of the year that my father was perhaps terminally ill, I decided to try to complete it in time for the June graduation so that I could go home with a diploma while my father was still alive. I did in the end accomplish this objective, but it would never have been possible without the kind help of Prof. Christ. I imposed on him to read a draft of my thesis and comment on it in a far shorter time than a professor is normally required to do. Now that I have advised many thesis students myself, I have realized how much sacrifice he had to make to meet the deadline I forced upon him. My father remained conscious and alert for one week after I came home and died a week later. My thesis was eventually published as Chapters 1 and 6 in this collection.

A memorable occasion during my last year at Hopkins was the annual economists' meetings held that year in Boston. I was interviewed by more than twenty professors representing ten universities. (I believe this was about an average for a graduate student in the job market.) I have never experienced such hectic three days in my life. For Stanford I was interviewed by Marc Nerlove, who later phoned me from Stanford and offered me a position.

In the summer of 1964, I spent three months in Tokyo attending to my father's funeral and other related businesses and joined the Stanford faculty in September. During my second year at Stanford I received an offer of a lecturer's position from Hitotsubashi University in Tokyo thanks to the help of Prof. Tsuru, whom I mentioned earlier. I accepted the offer because a position in a major university in Japan becomes available only infrequently. In the fall of 1966, I resigned from Stanford and went to Hitotsubashi. For various reasons, however, I went back to Stanford in the fall of 1968 as an associate professor with tenure. During the two years in Japan, I met Yoshiko Miyaki, a Japanese literature major at Ferris Women's College in Yokohama. We got married in May 1969 and she joined me in Stanford.

In 1968, the year I went back to Stanford, Ted Anderson left Columbia and joined Stanford as professor of statistics and economics. He kindly suggested we should apply for a research grant from the National Science Foundation together, and from that time on we shared NSF research grants for many years. It was a very beneficial collaboration for me; on numerous occasions Ted offered me invaluable advice on statistical theory.

During the first several years of my research career, I worked mainly on the statistical problems of time series. Some of the time series papers I wrote in this period are collected in Part I of this volume. One day in 1972 my colleague Michael Boskin came to my office and asked me about statistical problems arising in the model James Tobin proposed in his 1958 *Econometrica* article. I was fascinated by the problems and the subsequent research led to Chapter 28 in this volume. This was a turning point of my career; since then my research interests have shifted away from time series toward limited and qualitative dependent variables models, represented by Parts III and IV of this volume.

In the meantime, our daughter Naoko was born in 1970 and our son Kenta in 1973. This book is dedicated to my family.