SOME SPECIFICATION TESTS

November 9, 2005

1. Poolability Tests for One-Way Fixed Effects

At the end of the "Analysis of Covariance" handout from October 19, I defined three F statistics to test "poolability," that is, constraints applied to all of one or more categories of fixed effects. Unfortunately, the degrees of freedom were wrong—sorry! So let's reconsider such tests, first in the fixed effects context, and then in the random effects context. Start with the most generic model, as in the beginning of Hsiao's chapter 2:

$$y_{it} = \alpha_{it} + x_{it}'\beta_{it} + u_{it} \tag{1.1}$$

Here x_{it} and β_{it} are $k \times 1$ vectors. We previously noted that this model cannot be estimated (at least not while we regard effects as fixed), since it has more parameters $(NT(K+1))$ than degrees of freedom (NT). One way to impose identification restrictions is to assume that all the parameters are constant over time. (This is not the only way to proceed, of course. For instance, one could instead assume constancy over space (i.e. cross-sectional units) and then test time-constancy restrictions.)

These assumed constraints give us:

$$y_{it} = \alpha_i + x_{it}'\beta_i + u_{it} \tag{1.2}$$

Now we have only $N(k+1)$ parameters, which means the model can be estimated provided that $T > (k+1)$, in which case we have $N(T-(k+1))$ degrees of freedom for estimating this "unrestricted" model. The simplest, most natural tests for pooling involve restrictions of constancy across space for *all* of the intercepts, slopes, or both.

As in Section 3 of the ANCOVA handout, a null hypothesis is that both slopes and intercepts are constant:

$$H_0 : \alpha_1 = \alpha_2 = \ldots = \alpha_N \text{ and } \beta_1 = \beta_2 = \ldots = \beta_N.$$

That hypothesis corresponds to imposing $(N-1)(k+1)$ restrictions on (1.2), to give:

$$y_{it} = \alpha + x_{it}'\beta + u_{it} \tag{1.3}$$

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09.11.05 (1)

A first alternative hypothesis maintains only the assumption of constant slopes, while allowing intercepts to vary:

$$H_1 : \alpha_i \neq \alpha_j \text{ for some } i \text{ and } j \text{ and } \beta_1 = \beta_2 = \ldots = \beta_N.$$

This hypothesis corresponds to imposing $(N-1)k$ restrictions on (1.2) to give:

$$y_{it} = \alpha_i + x'_{it}\beta + u_{it} \tag{1.4}$$

Or, equivalently, one can get (1.3) by imposing on (1.4) an additional $(N-1)$ restrictions representing equality of all the alphas.

A second alternative hypothesis is that slopes vary, but not intercepts:

$$H_2 : \alpha_1 = \alpha_2 = \ldots = \alpha_N \text{ and } \beta_j \neq \beta_k \text{ for some} j, k.$$

This hypothesis corresponds to imposing $(N-1)$ restrictions on (1.2) to give:

$$y_{it} = \alpha + x'_{it}\beta_i + u_{it} \tag{1.5}$$

Or, equivalently, by imposing an additional $(N-1)k$ restrictions on (1.5), that is, equality of all the betas, one can get (1.3). Hsiao ignores this possibility, and I cannot recall having seen an application in which this test seemed plausible.

Still another alternative hypothesis is that both intercepts and slopes vary across units:

$$H_3 : \alpha_i \neq \alpha_j \text{ and } \beta_k \neq \beta_l \text{ for some } i, j, k \text{ and } l.$$

This hypothesis, of course, corresponds to (1.2).

It will be assumed that the matrix of regressors is non-stochastic and of full column rank and that u_{it} is i.i.d. $N(0, \sigma^2)$.

With each hypothesis, one can associate an error sum of squares:

$$\text{SSE}_j = \sum_{i=1}^{N}\sum_{j=1}^{T} \hat{u}^2_{it(j)} \quad (j = 0, 1, 2, 3)$$

where $\hat{u}_{it(j)}$ is the least squares residual obtained by imposing H_j:

$$\hat{u}_{it(j)} = y_{it} - \hat{\alpha}_{i(j)} - x'_{it}\hat{\beta}_{i(j)}$$

Consider first H_0. To obtain $\hat{u}_{(0)}$, regress y on X and a constant, then use Proposition 1 from the ANCOVA handout (i.e. the two-step estimation result):

$$\hat{\beta}_{i(0)} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T} (x_{it} - \bar{x}_{..})(x_{it} - \bar{x}_{..})' \right)^{-1} (x_{it} - \bar{x}_{..}) y_{it} \equiv \hat{\beta}_{(0)}$$

$$\hat{\alpha}_{i(0)} = \bar{y}_{..} - \bar{x}'_{..}\hat{\beta}_{(0)}$$

It follows that:

$$\hat{u}_{it(0)} = (y_{it} - \bar{y}_{..}) - (x_{it} - \bar{x}_{..})'\hat{\beta}_{(0)} \tag{1.6}$$

Next, under H_1, Proposition 2 gives $\hat{\beta}_{i(1)} = \beta_W$ and $\hat{\alpha}_{i(1)} = \bar{y}_{i.} - \bar{x}'_{i.}\hat{\beta}_W$, so:

$$\hat{u}_{it(1)} = (y_{it} - \bar{y}_{i.}) - (x_{it} - \bar{x}_{i.})'\hat{\beta}_W = \tilde{y}_{it} - \tilde{x}'_{it}\hat{\beta}_W \tag{1.7}$$

which are the residuals from the within-groups regression.

The coefficients in the unrestricted regression (1.2) can be estimated by applying least squares separately to each of the N sets of T observations, yielding:

$$\hat{\beta}_{i(3)} = \left(\sum_{t=1}^{T}(x_{it} - \bar{x}_{i.})(x_{it} - \bar{x}_{i.})'\right)^{-1} \sum_{t=1}^{T}(x_{it} - \bar{x}_{i.})(y_{it} - \bar{y}_{i.})$$

$$\hat{\alpha}_{i(3)} = \bar{y}_{i.} - \bar{x}'_{i.}\hat{\beta}_{(3)}$$

Upon substitution:

$$\hat{u}_{it(2)} = (y_{it} - \bar{y}_{i.}) - (x_{it} - \bar{x}'_{i.}\hat{\beta}_{i(2)}) \tag{1.8}$$

The residual sums of squares can be computed using (1.6), (1.7), and (1.8). In turn, we can form an F statistic to test H_0 against H_3 as follows:

$$F_3 = \frac{(SSE_0 - SSE_3)/((N-1)(k+1))}{SSE_3/(N(T-(k+1))}$$

where we make use of the fact that under H_0, F_3 follows an F distribution with $(N-1)(k+1)$ numerator d.f. and $NT - Nk - N$ denominator d.f. (This is Hsiao's F_3, but what I'm labelling SSE_0 is his S_3, my SSE_3 is his S_1, and my H_0 is his H_3.)

To test H_1 against H_3, form:

$$F_1 = \frac{(SSE_1 - SSE_3)/((N-1)k)}{SSE_3/(N(T-(k+1))}$$

where we make use of the fact that under H_1, F_1 follows an F distribution with $(N-1)k$ numerator d.f. and $NT - Nk - N$ denominator d.f. (This is Hsiao's F_1, and this time our H_1s agree (but my notation for F statistics was inconsistent.))

At the outset, we assumed, without testing, constancy over time. We could instead have assumed constancy across units, and then tested for constant parameters over time. For instance, we could have found the SSE from running 20 different OLS models each having 10 observations and 3 parameters (and thus only 7 degrees of freedom). Then we could then have computed a test statistic that would be distributed as $F(57, 140)$, to test the 57 restrictions imposed on these models to yield a single OLS regression for the whole sample. In this case, the dimensions of the panel weakly favour the opposite approach, since $T > N$. With very small N or T, one approach or the other is likely to be infeasible.

2. Poolability in Two-Way Fixed Effects Extending the F tests in section 1 to two-way models is fairly simple, since the difference between one-way and two-way fixed effects models is simply a set of restrictions on parameters exactly like the restrictions we were just testing. Thus, given a two-way model with constant parameters:

$$y_{it} = \alpha_i + \eta_t + x'_{it}\beta + u_{it} \tag{2.1}$$

one can test the T restrictions that generate the one-way model:

$$y_{it} = \alpha_i + x'_{it}\beta + u_{it} \tag{2.2}$$

with an F test based on SSE of the unrestricted (2.1) and the restricted (2.2). A difficulty–which is generic to specification tests, not unique to the panel data context–is that there are huge numbers of possible restrictions, that can be nested in huge numbers of ways. Thus far, for simplicity, we have considered only tests that restrict whole sets of parameters simultaneously. Suppose that we start with:

$$y_{it} = \alpha_i + \eta_t + x'_{it}\beta_t + u_{it} \tag{2.3}$$

and assume that we want to test for change points, which multiple runs of βs that are equal. Even if we know that we expect only one change point, there are $T-2$ restricted models to examine (fewer if we set a minimum number of observations per "regime"). If we don't allow for an unknown number of change points, the number of possible restricted models grows exponentially, even before we bring in possible restrictions on the fixed effects.

There is a very large literature on change points, some of which can be adapted to the panel data context, but we will not wade into this topic in our remaining weeks. Obviously, between the extremes of estimating all parameters for the model separately by spatial unit (or time unit) and estimating a pooled model with no time or space effects, there are many intermediate assumptions. In chapter 6, Hsiao discusses a variety of varying-coefficient models. Baltagi doesn't really try to include the topic.

2. Poolability Tests for Random Effects Perhaps surprisingly, the same basic framework as in Section 1 works to test poolability under the random effects, or error components, perspective as well. The key step in generating random effects estimates was estimating the variance components, and then using those to construct weights such that WLS would give the GLS estimates we sought. Since the estimation, after weighting, is just by OLS, the procedure from Section 1 carries over. Skipping the technical details, the F test is a special case of a procedure outlined by Roy and Zellner, so the random effects version of the F test described above is called the "Roy-Zellner test" for poolability. Since Ω and Σ are not actually known, but estimated, the test statistic only approximately follows an F distribution.

These F tests are not the last word on pooling. In particular, there might be reason to trade off lower variance for some bias, in which case some other criterion like MSE would be necessary. As usual, there's a not-small, rather technical literature to explore, but we'll not wade into it right now.

Grunfeld Data:one-way model poolability (F) tests

H_0: $\alpha_i=\alpha_j$, $\beta_k=\beta_l$ for all $i,j; k,l$ vs.
H_3: $\alpha_i\neq\alpha_j$, $\beta_k\neq\beta_l$ for some $i,j; k,l$

[unrestricted estimates — estimate coefficients for each firm in separate model]

```
. reg i f c if(firm==1)

      Source |       SS       df       MS              Number of obs =      20
-------------+------------------------------           F(  2,    17) =   99.58
       Model |  1677686.65     2  838843.326           Prob > F      =  0.0000
    Residual |   143205.85    17  8423.87352           R-squared     =  0.9214
-------------+------------------------------           Adj R-squared =  0.9121
       Total |   1820892.5    19  95836.4475           Root MSE      =  91.782

------------------------------------------------------------------------------
           i |      Coef.   Std. Err.      t    P>|t|      [95% Conf. Interval]
-------------+----------------------------------------------------------------
           f |   .1192808   .0258342     4.62   0.000       .0647755    .1737861
           c |   .3714448   .0370728    10.02   0.000        .293228    .4496616
       _cons |  -149.7824   105.8421    -1.42   0.175      -373.0897    73.52493
------------------------------------------------------------------------------

. reg i f c if(firm==2)

      Source |       SS       df       MS              Number of obs =      20
-------------+------------------------------           F(  2,    17) =    7.56
       Model |  140682.043     2  70341.0214           Prob > F      =  0.0045
    Residual |  158093.277    17  9299.60455           R-squared     =  0.4709
-------------+------------------------------           Adj R-squared =  0.4086
       Total |   298775.32    19  15725.0168           Root MSE      =  96.434

------------------------------------------------------------------------------
```

```
           i |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
-------------+------------------------------------------------------------------
           f |    .174856    .0741981      2.36    0.031       .0183118    .3314002
           c |   .3896419    .1423669      2.74    0.014        .089274    .6900097
       _cons |  -49.19831    148.0754     -0.33    0.744        -361.61    263.2134
--------------------------------------------------------------------------------
```

[etc. through (firm==10)]

```
. reg i f c if(firm==10)

      Source |       SS       df       MS              Number of obs =      20
-------------+------------------------------           F(  2,    17) =   15.32
       Model |  36.0953645     2  18.0476822           Prob > F      =  0.0002
    Residual |   20.026731    17    1.178043           R-squared     =  0.6432
-------------+------------------------------           Adj R-squared =  0.6012
       Total |  56.1220955    19   2.9537945           Root MSE      =  1.0854

--------------------------------------------------------------------------------
           i |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
-------------+------------------------------------------------------------------
           f |   .0045734    .0271608      0.17    0.868      -.0527308    .0618777
           c |   .4373692    .0795889      5.50    0.000       .2694513    .6052871
       _cons |   .1615187    2.065564      0.08    0.939      -4.196441    4.519478
--------------------------------------------------------------------------------
```

yields $\mathbf{SSE_3}$=143,205.85 + 158,093.28 + 13,216.59 + 2,997.44 + 1,396.84 + 1,110.53 + 1,507.40 + 1,773.23 + 1,407.36 + 20.026731 = **324,728.55**

[pool intercepts and slopes with one OLS model for whole sample, no f.e.]

```
. reg i f c

      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F(  2,   197) =  426.58
       Model |  7604093.48     2  3802046.74           Prob > F      =  0.0000
    Residual |  1755850.43   197  8912.94636           R-squared     =  0.8124
-------------+------------------------------           Adj R-squared =  0.8105
       Total |  9359943.92   199  47034.8941           Root MSE      =  94.408

------------------------------------------------------------------------------
           i |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           f |   .1155622   .0058357    19.80   0.000     .1040537    .1270706
           c |   .2306785   .0254758     9.05   0.000     .1804382    .2809188
       _cons |  -42.71437   9.511676    -4.49   0.000    -61.47215   -23.95659
------------------------------------------------------------------------------
```

So $\mathbf{SSE_0}$**= 1,755,850.43**

We had F_3=[SSE_0-SSE_3/(N-1)(k+1)]/[SSE_3/(N(T-(k+1))]

= (1,755,850.43 – 324,728.55)/27)/(324,728.55/170) = **27.75**

```
. display F(27,170,27.75)
        1
```

so **p=0.000, i.e. we reject poolability of all coefficients ($\mathbf{H_0}$)**

[COMPARE TO BALTAGI, 3RD ED., PAGE 57]

H_1: $\alpha_i = \alpha_j$, $\beta_k \neq \beta_l$ for all $i,j;$ for some k,l vs.
H_3: $\alpha_i \neq \alpha_j$, $\beta_k \neq \beta_l$ for some $i,j;\ k,l$

```
. anova i f c firm, continuous(f c)
```

Number of obs = 200 | R-squared = 0.9441
Root MSE = 52.768 | Adj R-squared = 0.9408

Source	Partial SS	df	MS	F	Prob > F
Model	8836465.8	11	803315.073	288.50	0.0000
f	240201.566	1	240201.566	86.27	0.0000
c	888838.4	1	888838.4	319.21	0.0000
firm	1232372.32	9	136930.258	49.18	0.0000
Residual	**523478.114**	188	2784.45805		
Total	9359943.92	199	47034.8941		

Note that default ANCOVA table F-tests from STATA also test hypotheses on $\beta=0$ so they aren't equivalent to the F we defined to evaluate poolability of slopes allowing for variable intercepts. Instead, form:

$F_1=[SSE_1-SSE_3/(N-1)k]/[SSE_3/(N(T-(k+1))]$

$= (523{,}478.11 - 324{,}728.55)/18)/(324{,}728.55/170) = \mathbf{5.78}$

a smaller F, but still **p=0.000**, so we can reject poolability of slopes with varying intercepts

p. 58 "Note that one could have tested poolability across time. The Chow test gives an observed value of 1.12 which is distributed as $F(57,140)$. This does not reject poolability across time…"

```
.gen urss=0
.foreach y of numlist 1935/1954 {
    2. reg i f c if[year==`y']
    3. replace urss=urss+e(rss)
    4. }

      Source |       SS       df       MS              Number of obs =      10
-------------+------------------------------           F(  2,      7) =   22.48
       Model |  84964.8462     2  42482.4231           Prob > F      =  0.0009
    Residual |  13230.6643     7   1890.0949           R-squared     =  0.8653
-------------+------------------------------           Adj R-squared =  0.8268
       Total |  98195.5105     9  10910.6123           Root MSE      =  43.475

------------------------------------------------------------------------------
           i |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           f |   .1024979   .0157931     6.49   0.000     .0651532    .1398425
           c |  -.0019948   .2148591    -0.01   0.993    -.5100557    .5060662
       _cons |   .3560334   23.82794     0.01   0.988    -55.98808    56.70015
------------------------------------------------------------------------------
(200 real changes made)
(ETC.)

. list urss in 1
     +-----------+
     |      urss |
     |-----------|
  1. |  1205818  |
     +-----------+
```

F=((1,755,850.43 – 1,205,818)/57)/(1,205,818/140) = **1.12 (p=0.29)**

In a random effects context, we can perform exactly the same tests, but with WLS instead of OLS, with the weights determined by estimates of the error components. I previously demonstrated calculation of thetas (the weighting parameters) and then FGLS estimation using these thetas.

[canned random effects (Swamy-Arora) does not output SSE]

```
. xtreg i f c, i(firm) sa

Random-effects GLS regression                   Number of obs      =       200
Group variable (i): firm                        Number of groups   =        10

R-sq:  within  = 0.7668                         Obs per group: min =        20
       between = 0.8196                                        avg =      20.0
       overall = 0.8061                                        max =        20

Random effects u_i ~ Gaussian                   Wald chi2(2)       =    657.67
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000

------------------------------------------------------------------------------
           i |      Coef.   Std. Err.       z     P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           f |   .1097811   .0104927    10.46   0.000      .0892159    .1303464
           c |    .308113   .0171805    17.93   0.000      .2744399    .3417861
       _cons |  -57.83441   28.89893    -2.00   0.045     -114.4753   -1.193537
-------------+----------------------------------------------------------------
     sigma_u |   84.20095
     sigma_e |  52.767964
         rho |  .71800838   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

[we could generate SSE or we could just run WLS, given θ] [for Swamy-Arora, estimated θ=0.861224 (2nd ed. and p. 22, 3rd ed.)]

```
. gen isw=(i-0.861224*ib)

. gen csw=(c-0.861224*cb)

. gen fsw=(f-0.861224*fb)

. reg isw fsw csw

      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F(  2,   197) =  328.84
       Model |  1832485.78     2  916242.888           Prob > F      =  0.0000
    Residual |  548903.934   197  2786.31439           R-squared     =  0.7695
-------------+------------------------------           Adj R-squared =  0.7672
       Total |  2381389.71   199  11966.7825           Root MSE      =  52.786

------------------------------------------------------------------------------
         isw |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         fsw |   .1097811   .0104927    10.46   0.000     .0890888    .1304735
         csw |    .308113   .0171805    17.93   0.000     .2742318    .3419942
       _cons |  -8.026028   4.010488    -2.00   0.047    -15.93503   -.1170292
------------------------------------------------------------------------------
```

So $F=[SSE_0-SSE_3/(N-1)(k+1)]/[SSE_3/(N(T-(k+1))]$

$$= (548{,}903.93 - 324{,}728.55)/27)/(324{,}728.55/170) = \mathbf{4.35}$$

```
. display F(27,170,4.35)
1
```

So we reject poolability of all coefficients even when allowing for one-way error component disturbances. [Compare Baltagi 3rd ed., page 58, 'Roy-Zellner']

Two-Way or One-Way Fixed Effects?

```
. anova i f c firm year, continuous(f c) reg nocons

      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F( 31,   169) =  158.78
       Model |   13168559     31  424792.227           Prob > F      =  0.0000
    Residual | 452147.043    169  2675.42629           R-squared     =  0.9668
-------------+------------------------------           Adj R-squared =  0.9607
       Total | 13620706.1    200  68103.5304           Root MSE      =  51.725

------------------------------------------------------------------------------
           i       Coef.   Std. Err.       t    P>|t|     [95% Conf. Interval]
------------------------------------------------------------------------------
f               .1177158   .0137513     8.56   0.000      .0905694    .1448623
c               .3579163    .022719    15.75   0.000      .3130667    .4027659
firm
           1   -180.4264   65.00055    -2.78   0.006      -308.744   -52.10879
           2    26.62781   36.91653     0.72   0.472     -46.24913    99.50474
           3   -315.6572   37.07073    -8.51   0.000     -388.8386   -242.4759
           4    -85.0726   25.05781    -3.40   0.001     -134.5392   -35.60598
           5    -185.865   27.22086    -6.83   0.000     -239.6018   -132.1283
           6   -77.53781   23.48448    -3.30   0.001     -123.8985   -31.17708
           7   -128.9598   24.81305    -5.20   0.000     -177.9433   -79.97639
           8   -112.9359   24.70425    -4.57   0.000     -161.7046   -64.16725
           9   -150.2089    24.9983    -6.01   0.000      -199.558   -100.8597
          10   -53.58933   21.59303    -2.48   0.014     -96.21613   -10.96253
year
           1    93.52622   27.10786     3.45   0.001      40.01258    147.0399
           2    74.32881   26.28383     2.83   0.005       22.4419    126.2157
           3    52.83621   26.07413     2.03   0.044      1.363259    104.3092
           4    54.29982    26.0657     2.08   0.039      2.843504    105.7561
           5    24.05593   25.52497     0.94   0.347     -26.33292    74.44478
           6    49.29115    25.4568     1.94   0.055     -.9631395    99.54543
           7    74.72176   25.33737     2.95   0.004      24.70324    124.7403
           8    72.38643   25.49756     2.84   0.005      22.05169    122.7212
           9     50.5486   25.17805     2.01   0.046      .8446089    100.2526
```

10	50.42746	25.13577	2.01	0.046	.8069269	100.048
11	37.84319	24.95347	1.52	0.131	-11.41746	87.10384
12	62.35694	24.80931	2.51	0.013	13.38087	111.333
13	54.13398	24.7417	2.19	0.030	5.291383	102.9766
14	49.80971	24.63549	2.02	0.045	1.176774	98.44264
15	20.03112	24.43391	0.82	0.413	-28.20387	68.26612
16	17.63011	24.19119	0.73	0.467	-30.12572	65.38595
17	31.04532	23.67891	1.31	0.192	-15.69923	77.78987
18	28.89389	23.43739	1.23	0.219	-17.37387	75.16166
19	25.80826	23.22233	1.11	0.268	-20.03496	71.65148
20	(dropped)					

```
. anova i f c firm, continuous(f c) reg nocons
```

Source	SS	df	MS
Model	13097228	12	1091435.66
Residual	**523478.114**	188	2784.45805
Total	13620706.1	200	68103.5304

Number of obs =	200
F(12, 188) =	391.97
Prob > F =	0.0000
R-squared =	0.9616
Adj R-squared =	0.9591
Root MSE =	52.768

i	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
f	.1101238	.0118567	9.29	0.000	.0867345	.1335131
c	.3100653	.0173545	17.87	0.000	.2758308	.3442999
firm						
1	-70.29669	49.70796	-1.41	0.159	-168.3537	27.76034
2	101.9058	24.93832	4.09	0.000	52.71093	151.1007
3	-235.5718	24.43162	-9.64	0.000	-283.7672	-187.3765
4	-27.80929	14.07775	-1.98	0.050	-55.57995	-.038631
5	-114.6168	14.16543	-8.09	0.000	-142.5604	-86.67319
6	-23.16129	12.66874	-1.83	0.069	-48.15244	1.829856
7	-66.55347	12.84297	-5.18	0.000	-91.88833	-41.21862
8	-57.54565	13.99315	-4.11	0.000	-85.14941	-29.9419

9	-87.22227	12.89189	-6.77	0.000	-112.6536	-61.79091
10	-6.567843	11.82689	-0.56	0.579	-29.89831	16.76262

$F = (SSE_R - SSE_U/T-1)) / (SSE_U/(NT-N-k-T+1))$

$= ((523,478.11-452,147.04)/19) / (452,147.04/179) = 1.49$

```
. display F(19,179,1.49)
.9065414
```

So we do not reject the restriction of no time fixed effects, given space fixed effects.

incumbency indicators

$Y_{it} = \alpha_i + \eta_t + X_{it}\beta$

test poolability if β's

Table 2

Estimated Incumbency Effects on Conservative Percentage of Vote, 1950-59

(vote percentages; standard errors in parentheses; n=2424)

	Models (continued below)...				
	(1)	(2)	(3)	(4)	(5)
Conservative Incumbents					
General, Same	1.43 (0.38)	1.45 (0.38)	1.00 (0.36)		
General, New	-0.05 (0.49)	-0.13 (0.48)			
By-, Same	0.85 (0.73)	0.99 (0.64)	0.95 (0.64)	→	↘
By-, New	1.37 (1.16)				
General or By-, Same				1.35 (0.37)	0.99 (0.35)
General or By-, New				0.08 (0.47)	
Labour Incumbents					
General, Same	-0.73 (0.42)	-0.67 (0.42)	-0.97 (0.41)		
General, New	-1.66 (0.51)	-1.70 (0.50)			
By-, Same	-1.61 (0.73)	-1.04 (0.55)	-0.82 (0.55)	→	↘
By-, New	-0.49 (0.71)				
General or By-, Same				-0.74 (0.41)	-0.93 (0.40)
General or By-, New				-1.40 (0.48)	
Liberal (all)	-4.19 (1.84)	-4.12 (1.84)	-4.41 (1.84)	-4.10 (1.84)	-4.40 (1.84)
R^2	0.958	0.957	0.957	0.957	0.957
F_1		1.18	5.64	2.56	3.78
p value		0.307	0.000	0.037	0.001

	...Models (continued from above)				
	...	(6)	(7)	(8)	(9)
Conserv. (+) or Labour (-) Inc.s					
General, Same		1.05 (0.21)	0.98 (0.21)		
General, New		0.88 (0.28)			
By-, Same		1.19 (0.48)	0.87 (0.36)	→	↘
By-, New		0.42 (0.54)			
General or By-, Same				1.02 (0.21)	0.97 (0.20)
General or By-, New				0.79 (0.26)	
Liberal (all)		-4.38 (1.82)	-4.45 (1.82)	-4.43 (1.81)	-4.42 (1.81)
R^2		0.957	0.957	0.957	0.957
F_1		4.97	3.76	3.48	3.24
p value		0.001	0.001	0.002	0.002

The Breusch-Pagan LM Test

In the one-way model, if var(μ_i) = $\sigma_\mu^2 = 0$ then the RE are superfluous. So BP devised an LM test for H_0: $\sigma_\mu^2 = 0$. In the two-way model, the RE are superfluous if both $\sigma_\mu^2 = 0$ and $\sigma_\lambda^2 = 0$.

Derivation of the statistic is quite involved (to begin with, it is formulated in terms of the MLE estimator of the variance components, which we have (thus far) skipped), so I won't attempt to work through the math. But the key results are that:

$$LM_1 = \frac{NT}{2(T-1)}\left[1-\frac{\tilde{u}'(I_N \otimes J_T)\tilde{u}}{\tilde{u}'\tilde{u}}\right]^2, \quad LM_2 = \frac{NT}{2(N-1)}\left[1-\frac{\tilde{u}'(J_N \otimes I_T)\tilde{u}}{\tilde{u}'\tilde{u}}\right]^2$$

and $LM = LM_1 + LM_2$ where the u-tildes are the OLS residuals

Under H_0, LM ~ χ_2^2

However, BP uses two-sided alternative even though variances must be positive. Honda suggests:

$$HO = \sqrt{\frac{NT}{2(T-1)}}\left[\frac{\tilde{u}'(I_N \otimes J_T)\tilde{u}}{\tilde{u}'\tilde{u}} - 1\right] \xrightarrow{H_0^a} N(0,1)$$

Breusch-Pagan LM Test on Grunfeld data

one-way random effects (space), $H_0^a: \sigma_\mu = 0$

```
. xtreg i f c, i(firm) re

Random-effects GLS regression                   Number of obs      =       200
Group variable (i): firm                        Number of groups   =        10

R-sq:  within  = 0.7668                         Obs per group: min =        20
       between = 0.8196                                        avg =      20.0
       overall = 0.8061                                        max =        20

Random effects u_i ~ Gaussian                   Wald chi2(2)       =    657.67
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000

------------------------------------------------------------------------------
           i |      Coef.   Std. Err.       z     P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           f |   .1097811   .0104927     10.46   0.000      .0892159    .1303464
           c |    .308113   .0171805     17.93   0.000      .2744399    .3417861
       _cons |  -57.83441   28.89893     -2.00   0.045     -114.4753   -1.193537
-------------+----------------------------------------------------------------
     sigma_u |   84.20095
     sigma_e |  52.767964
         rho |  .71800838   (fraction of variance due to u_i)
------------------------------------------------------------------------------

. xttest0

Breusch and Pagan Lagrangian multiplier test for random effects:

        i[firm,t] = Xb + u[firm] + e[firm,t]
```

```
Estimated results:
                 |       Var     sd = sqrt(Var)
        ---------+-----------------------------
               i |   47034.89       216.8753
               e |   2784.458       52.76796
               u |     7089.8       84.20095

Test:   Var(u) = 0
                      chi2(1) =   798.16
                  Prob > chi2 =     0.0000
```

one-way random effects (time), $H_0^b: \sigma_\eta = 0$

```
. xtreg i f c, i(year) re

Random-effects GLS regression                   Number of obs      =       200
Group variable (i): year                        Number of groups   =        20

R-sq:  within  = 0.8037                         Obs per group: min =        10
       between = 0.9347                                        avg =      10.0
       overall = 0.8124                                        max =        10

Random effects u_i ~ Gaussian                   Wald chi2(2)       =    853.15
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000

------------------------------------------------------------------------------
           i |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           f |   .1155622   .0058357    19.80   0.000     .1041244    .1269999
           c |   .2306785   .0254758     9.05   0.000     .1807468    .2806101
       _cons |  -42.71437   9.511676    -4.49   0.000    -61.35691   -24.07183
-------------+----------------------------------------------------------------
     sigma_u |          0
     sigma_e |  98.099115
         rho |          0   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

```
. xttest0

Breusch and Pagan Lagrangian multiplier test for random effects:

        i[year,t] = Xb + u[year] + e[year,t]

        Estimated results:
                         |       Var     sd = sqrt(Var)
                ---------+-----------------------------
                       i |   47034.89       216.8753
                       e |   9623.436       98.09912
                       u |          0              0

        Test:   Var(u) = 0
                             chi2(1) =     6.45
                         Prob > chi2 =     0.0111
```

two-way random effects (space & time), $H_0^c\!:\sigma_\mu = \sigma_\eta = 0$

$798.16 + 6.45 = 804.61 \sim \chi_2^2$ so p=0, and we reject all H_0s

[above replicates row 1 of Table 4.1 plus Table 4.2 in 3rd ed. of Baltagi]

[For kicks, let's use R to check up on STATA's canned routines]

```
> grun <- read.csv("c:/documents/teaching/493Panel/grfldat.csv")
> y <- grun[,3]
> f <- grun[,4]
> c <- grun[,5]
> X <- grun[,4:5]
> X<- cbind(1,X)
>
> lm.OLS<-lm(y~f+c)
>
> e<- resid(lm.OLS)
> IN<-matrix(0,10,10)
> JT<-matrix(1,20,20)
>
> diag(IN)<-1
> M<-kronecker(IN,JT)
> LM1<-200/38*(1-t(e)%*%M%*%e/t(e)%*%e)^2
> LM1

          [,1]
[1,] 798.1615
```

```
> JN<-matrix(1,10,10)
> IT<-matrix(0,20,20)
> diag(IT)<-1
> M2<-kronecker(JN,IT)
> LM2<-200/18*(1-t(e)%*%M2%*%e/t(e)%*%e)^2
> LM2

          [,1]
 [1,] 6.453882


> H1<-(200/38)^.5*(t(e)%*%M%*%e/t(e)%*%e-1)
> H1

          [,1]
 [1,] 28.25175
```

Critical value around 1.645 (one-sided)

∴ reject null (there are unit effects)

```
> H2<-(200/18)^.5*(t(e)%*%M2%*%e/t(e)%*%e-1)
> H2

           [,1]
 [1,] -2.540449
```

Critical value around 1.645 (one-sided)

∴ do **not** reject null (there are no time effects), & BP test misled us

See Baltagi, Chang, and Li for further refinements

Fixed or Random Effects?

previously, we estimated (one-way) random effects $I_{it} = \alpha + \beta_1 F_{it} + \beta_2 C_{it} + u_{it}$

w/ $u_{it} = \mu_i + v_{it}$

by FGLS, weighting with $\theta = 1 - \sqrt{\frac{\sigma_v^2}{T\sigma_\mu^2 + \sigma_v^2}}$

• if $\sigma_\mu^2=0$, $\theta=0$, there are no unit effects, and WLS is just OLS (as on the Baltagi table)

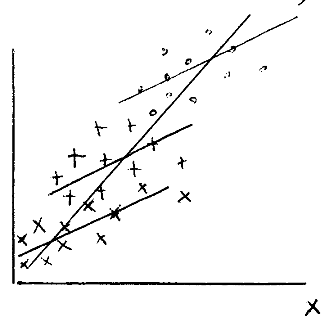
• if $\sigma_v^2=0$, $\theta=1$, and all the variation is within units (the within row shows $\theta=1$ on the Baltagi table)

• as T increases, θ goes to 0, and the FGLS model converges on the OLS model, and FE and RE differ less and less (Hsiao makes this point in section 3.4)

Whereas in ANOVA, we chose fixed or random effects on substantive grounds, now that we have covariates in addition to the space and time factors, the relationship between these covariates x_{it} and the effects (e.g the μ_is) is critical. The random effects model treats them as uncorrelated; fixed effects does not. Otherwise, the models are "the same" and differ only insofar as θ differs from 0.

Hausman Test Intuition

- RE estimation does not wipe out time-invariant regressors or consume d.f. voraciously as FE estimation does, but it is biased when X is correlated with μ_i



common slopes
variable intercepts

$X \overset{+}{\propto} \mu_i$

$\therefore$ errors $\overset{+}{\propto}$ X

given error components

→ bias

idea behind Hausman statistic: is RE estimate significantly different from unbiased FE estimate?

Digression / Reminder

Limits

$$\lim_{x \to a} f(x) = A \quad \text{or} \quad f(x) \to A \; \text{as} \; x \to a$$

means $\forall \varepsilon > 0 \;\; \exists \text{ a } \delta > 0 \ni$

$|f(x) - A| < \varepsilon$ if $x \in D_f$ and

$0 < |x - a| < \delta$

(D_f is domain of f)

Probability Limits

A necessary condition for consistency of estimator $\hat{\theta}_n$ (which is an estimator of population parameter θ) is that the ultimate distribution of $\hat{\theta}_n$ be degenerate at some point on the real line (i.e. converges to a single point).

The probability limit of $\hat{\theta}_n$ is the point on the real line to which it's sampling distribution converges as the sample size increases without limit.

- $\hat{\theta} \xrightarrow{P} \theta_0$ (read as "converges in probability to")
- $\text{plim } \hat{\theta} = \theta_0$
- $\underset{n \to \infty}{\text{plim}} \ \hat{\theta}_n = \theta_0$

if $P(|\hat{\theta}_n - \theta_0| \leq \varepsilon) = 1$ as $n \to \infty$ $\forall \ \varepsilon > 0$

<u>or</u> $\lim_{n \to \infty} P(-\varepsilon \leq \hat{\theta}_n - \theta_0 \leq \varepsilon) = 1$

HAUSMAN TEST

IF FE & RE MODELS ARE THE SAME EXCEPT FOR ASSUMPTIONS ABOUT $\text{cov}(\mu_i, x_{it})$, AND THEY DIFFER ONLY AS

$$\frac{\sigma_\nu^2}{T\sigma_\mu^2 + \sigma_\nu^2}$$ DIFFERS FROM 0 THEN

- UNDER H_0: $\text{cov}(\mu_i, x_{it}) = 0$, BOTH FE & RE ARE CONSISTENT, BUT RE MORE EFFICIENT

- UNDER H_A: $\text{cov}(\mu_i, x_{it}) \neq 0$, FE ESTIMATOR CONSISTENT, RE NOT

USE ~~FORM~~ $\hat{q}_1 = \hat{\beta}_{GLS} - \tilde{\beta}_W$

UNDER H_0. $\text{plim}\ \hat{q}_1 = 0$

$\text{cov}(\hat{q}_1, \hat{\beta}_{GLS}) = 0$

$$\hat{\beta}_{GLS} - \beta = (X'\Omega^{-1}X)^{-1} X'\Omega^{-1}u$$

$$\tilde{\beta}_W - \beta = (X'QX)^{-1} X'Qu \quad , \quad Q = I_N \otimes C_T$$

∴ Hausman test stat

$$m_1 = \hat{q}_1' [var(\hat{q}_1)]^{-1} \hat{q}_1 \overset{H_0}{\sim} \chi^2_K$$

We don't know Ω, so we use $\hat{\Omega}$ & $\hat{\beta}_{GLS}$ is actually $\hat{\beta}_{FGLS}$

Hausman & Taylor derive equivalent tests from comparisons using $\hat{\beta}_B$

$$\hat{q}_1 = \hat{\beta}_{GLS} - \tilde{\beta}_W$$

$$\hat{q}_2 = \hat{\beta}_{GLS} - \hat{\beta}_B$$

$$\hat{q}_3 = \tilde{\beta}_W - \hat{\beta}_B$$

$$m_i = \hat{q}_i' V_i^{-1} \hat{q}_i \qquad V_i = var(\hat{q}_i)$$

all $\overset{H_0}{\sim} \chi^2_K$

Baltagi added $\hat{q}_4 = \hat{\beta}_{GLS} - \hat{\beta}_{OLS}$

$$\underset{NT\times NT}{Q} = C_N \otimes C_T = \underset{NT\times NT}{I_N \otimes I_T} - I_N \otimes \bar{J}_T - \bar{J}_N \otimes I_T + \bar{J}_N \otimes \bar{J}_T$$

AND $\tilde{u} = Qu$ HAS ELEMENTS

$$\tilde{u}_{it} = (u_{it} - \bar{u}_{i.} - \bar{u}_{.t} + \bar{u}_{..})$$

(SO THIS $Q = W^*$ IN THE NERLOVE APPENDIX)

$\therefore E(\hat{q}_1) = 0$

$$\text{cov}(\hat{\beta}_{GLS}, \hat{q}_1) = \text{var}(\hat{\beta}_{GLS}) - \text{cov}(\hat{\beta}_{GLS}, \tilde{\beta}_W)$$

$$= (X'\Omega^{-1}X)^{-1} - (X'\Omega^{-1}X)^{-1} X\Omega^{-1} E(uu') QX(X'Q$$

$$= (X'\Omega^{-1}X)^{-1} - (X'\Omega^{-1}X)^{-1} = 0$$

$\tilde{\beta}_W = \hat{\beta}_{GLS} - \hat{q}_1$ SO

$$\text{var}(\tilde{\beta}_W) = \text{var}(\hat{\beta}_{GLS}) + \text{var}(\hat{q}_1)$$

since $\text{cov}(\hat{\beta}_{GLS}, \hat{q}_1) = 0$

$$\therefore \text{var}(\hat{q}_1) = \text{var}(\tilde{\beta}_W) - \text{var}(\hat{\beta}_{GLS})$$

$$= \sigma_v^2 (X'QX)^{-1} - (X'\Omega^{-1}X)^{-1}$$

Baltagi Table 2.1 (1st. ed)

	β_1	β_2	$\theta=1-(\sigma_\nu/\sigma_1)$
OLS	0.11556	0.23068	0
	(0.00584)	(0.02548)	
Between	**0.13465**	**0.03203**	∞
	(0.02875)	**(0.19094)**	
Within	**0.11012**	**0.31007**	1
	(0.01186)	**(0.01735)**	
WALHUS	0.10973	0.30765	0.845779
	(0.01029)	(0.01724)	
AMEMIYA	0.10979	0.30817	0.863097
	(0.01052)	(0.01717)	
SWAR	**0.10978**	**0.30811**	0.861224
	(0.01049)	**(0.01718)**	
NERLOVE	0.10978	0.30810	0.860717
	(0.01049)	(0.01718)	
IMLE	0.10976	0.30794	0.855359
	(0.01042)	(0.01720)	

Grunfeld Data: one-way model Hausman test

```
. xtreg i f c, i(firm) re

Random-effects GLS regression                   Number of obs      =       200
Group variable (i): firm                        Number of groups   =        10

R-sq:  within  = 0.7668                         Obs per group: min =        20
       between = 0.8196                                        avg =      20.0
       overall = 0.8061                                        max =        20

Random effects u_i ~ Gaussian                   Wald chi2(2)       =    657.67
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000

------------------------------------------------------------------------------
           i |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           f |   .1097811   .0104927    10.46   0.000     .0892159    .1303464
           c |    .308113   .0171805    17.93   0.000     .2744399    .3417861
       _cons |  -57.83441   28.89893    -2.00   0.045    -114.4753   -1.193537
-------------+----------------------------------------------------------------
     sigma_u |   84.20095
     sigma_e |  52.767964
         rho |  .71800838   (fraction of variance due to u_i)
------------------------------------------------------------------------------

. xthausman
(Warning:  xthausman is no longer a supported command; use -hausman-.  For instructions, see help hausman.)

Hausman specification test

                ---- Coefficients ----
             |      Fixed       Random
           i |     Effects      Effects       Difference
-------------+-----------------------------------------
           f |    .1101238     .1097811        .0003427
           c |    .3100653      .308113        .0019524

    Test:  Ho:  difference in coefficients not systematic

                 chi2(  2) = (b-B)'[S^(-1)](b-B), S = (S_fe - S_re)
                           =     2.33
                 Prob>chi2 =     0.3119
```

```
. xtreg i f c, fe i(firm)

Fixed-effects (within) regression               Number of obs      =       200
Group variable (i): firm                        Number of groups   =        10

R-sq:  within  = 0.7668                         Obs per group: min =        20
       between = 0.8194                                        avg =      20.0
       overall = 0.8060                                        max =        20

                                                F(2,188)           =    309.01
corr(u_i, Xb)  = -0.1517                        Prob > F           =    0.0000

------------------------------------------------------------------------------
           i |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           f |   .1101238   .0118567     9.29   0.000      .0867345    .1335131
           c |   .3100653   .0173545    17.87   0.000      .2758308    .3442999
       _cons |  -58.74393   12.45369    -4.72   0.000     -83.31086     -34.177
-------------+----------------------------------------------------------------
     sigma_u |  85.732501
     sigma_e |  52.767964
         rho |  .72525012   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0:     F(9, 188) =    49.18             Prob > F = 0.0000

. est store fixed

. xtreg i f c, i(firm) re

Random-èffects GLS regression                   Number of obs      =       200
Group variable (i): firm                        Number of groups   =        10

R-sq:  within  = 0.7668                         Obs per group: min =        20
       between = 0.8196                                        avg =      20.0
       overall = 0.8061                                        max =        20

Random effects u_i ~ Gaussian                   Wald chi2(2)       =    657.67
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000

------------------------------------------------------------------------------
           i |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           f |   .1097811   .0104927    10.46   0.000      .0892159    .1303464
           c |    .308113   .0171805    17.93   0.000      .2744399    .3417861
       _cons |  -57.83441   28.89893    -2.00   0.045     -114.4753   -1.193537
```

```
-------------+----------------------------------------------------------------
     sigma_u |   84.20095
     sigma_e |  52.767964
         rho |  .71800838   (fraction of variance due to u_i)
------------------------------------------------------------------------------

. hausman fixed .

                 ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |     fixed          .           Difference          S.E.
-------------+----------------------------------------------------------------
           f |    .1101238     .1097811        .0003427        .0055213
           c |    .3100653      .308113        .0019524        .0024516
------------------------------------------------------------------------------
                          b = consistent under Ho and Ha; obtained from xtreg
            B = inconsistent under Ha, efficient under Ho; obtained from xtreg

    Test:  Ho:  difference in coefficients not systematic

                  chi2(2) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                          =        2.33
                Prob>chi2 =      0.3119
```

We can compute the Hausman test statistic directly from the within and between estimates

```
xtreg i f c, i(firm) fe

Fixed-effects (within) regression               Number of obs      =       200
Group variable (i): firm                        Number of groups   =        10

R-sq:  within  = 0.7668                         Obs per group: min =        20
       between = 0.8194                                        avg =      20.0
       overall = 0.8060                                        max =        20

                                                F(2,188)           =    309.01
corr(u_i, Xb)  = -0.1517                        Prob > F           =    0.0000

------------------------------------------------------------------------------
           i |      Coef.   Std. Err.       t     P>|t|      [95% Conf. Interval]
-------------+----------------------------------------------------------------
           f |   .1101238   .0118567     9.29   0.000       .0867345    .1335131
           c |   .3100653   .0173545    17.87   0.000       .2758308    .3442999
       _cons |  -58.74393   12.45369    -4.72   0.000      -83.31086     -34.177
-------------+----------------------------------------------------------------
     sigma_u |  85.732501
     sigma_e |  52.767964
         rho |  .72525012   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0:     F(9, 188) =    49.18             Prob > F = 0.0000

. matrix define bw=e(b)

. matrix define Vw=e(V)

. matrix list bw

bw[1,3]
             f           c       _cons
y1    .1101238   .31006534  -58.743932

. matrix list Vw

symmetric Vw[3,3]
                f           c       _cons
    f   .00014058
    c  -.00007747   .00030118
_cons  -.13068152   .00066516   155.09442
```

```
. xtreg i f c, i(firm) be

Between regression (regression on group means)  Number of obs      =       200
Group variable (i): firm                        Number of groups   =        10

R-sq:  within  = 0.4778                         Obs per group: min =        20
       between = 0.8578                                        avg =      20.0
       overall = 0.7551                                        max =        20

                                                F(2,7)             =     21.11
sd(u_i + avg(e_i.))=  85.02366                  Prob > F           =    0.0011

------------------------------------------------------------------------------
           i |      Coef.   Std. Err.       t     P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           f |   .1346461   .0287455      4.68   0.002      .0666739    .2026183
           c |   .0320315   .1909378      0.17   0.872     -.4194647    .4835276
       _cons |  -8.527114   47.51531     -0.18   0.863      -120.883    103.8287
------------------------------------------------------------------------------

. matrix define bb=e(b)

. matrix define Vb=e(V)

. matrix list bb

bb[1,3]
             f           c       _cons
y1   .13464609   .03203148  -8.5271142


. matrix list Vb

symmetric Vb[3,3]
                f           c       _cons
    f    .0008263
    c  -.00370025   .03645724
_cons    .1275372  -6.0603362   2257.7044

. matrix define V=Vb+Vw

. matrix define q=bw-bb

. matrix list q

q[1,3]
```

```
                f           c        _cons
y1  -.02452229   .27803387  -50.216817

. matrix define m=q*syminv(V)*q'

. matrix list m

symmetric m[1,1]
           y1
y1  2.1313661


. gen m=trace(m)

. summarize m

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+-----------------------------------------------------
           m |       200    2.131366           0   2.131366   2.131366

. gen p=1-chi2(2,m)

. summariz p

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+-----------------------------------------------------
           p |       200    .3444924           0   .3444924   .3444924
```

Hausman Test (cont'd)

- large values → reject H_0: $\text{cov}(\alpha_i, x_{it}) = 0$ and do not implement RE

  - that does not mean FE necessarily correct

    Chamberlain outlines follow-up test based on reduced form regression involving all leads & lags

- since it involves FE, Hausman test doesn't use time-invariant regressors

- the test should fail as $T \to \infty$

  - also mis-specification can cause rejection of null so that properly specified RE would be OK

- I did Hausman test for one-way (two-way) model but the specification of different stats based on B, OLS, GLS, & W is more complicated for two-way (three-way)

- Kennedy describes easy way to implement H_0 as F-test (5th ed., p 312)